

Bridging Network and Circuit models: a low-rank approach

Croucher Computational Neuroscience

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Acknowledgements



Srdjan Ostojic
(Paris)

[Shao & SO 2023]

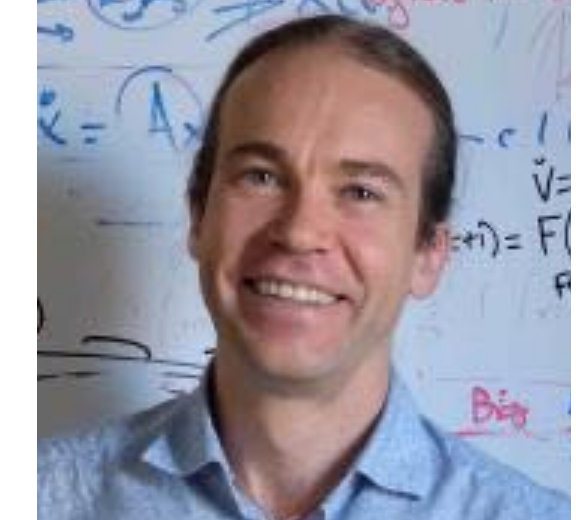


Stefano Recanatesi
(Haifa)

[Shao, Recanatesi, Dahmen, Shea-Brown & SO 2024]



David Dahmen
(Juelich)



Eric Shea-Brown
(Seattle)

Francesca Mastrogiuseppe (Lisbon)

[Mastrogiuseppe & SO 2018]

Friedrich Schuessler (Berlin)

[Schuessler et al 2020]

Daniel Marti (Seattle)

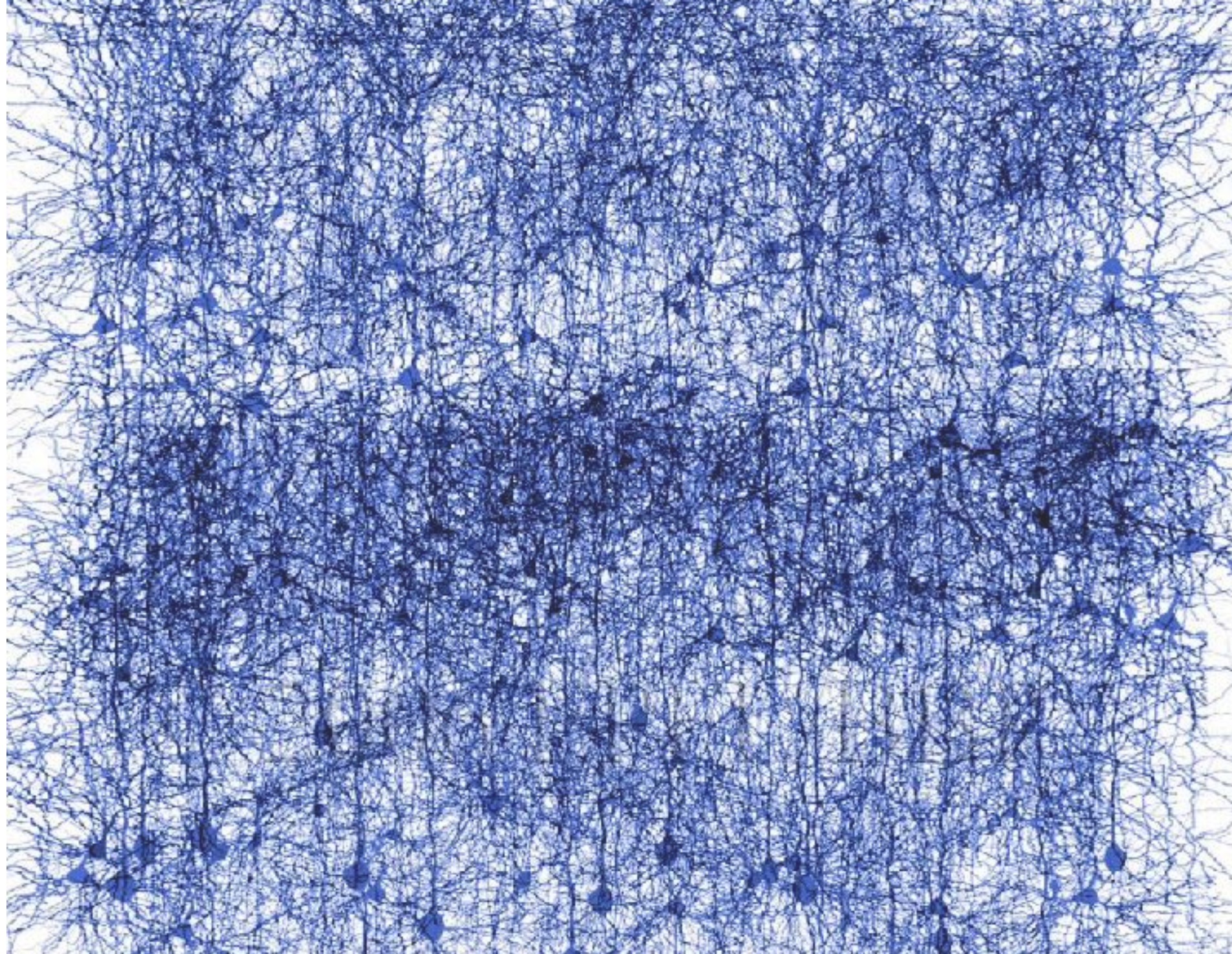
[Marti, Brunel & SO 2018]

Yu Hu (Hong Kong)

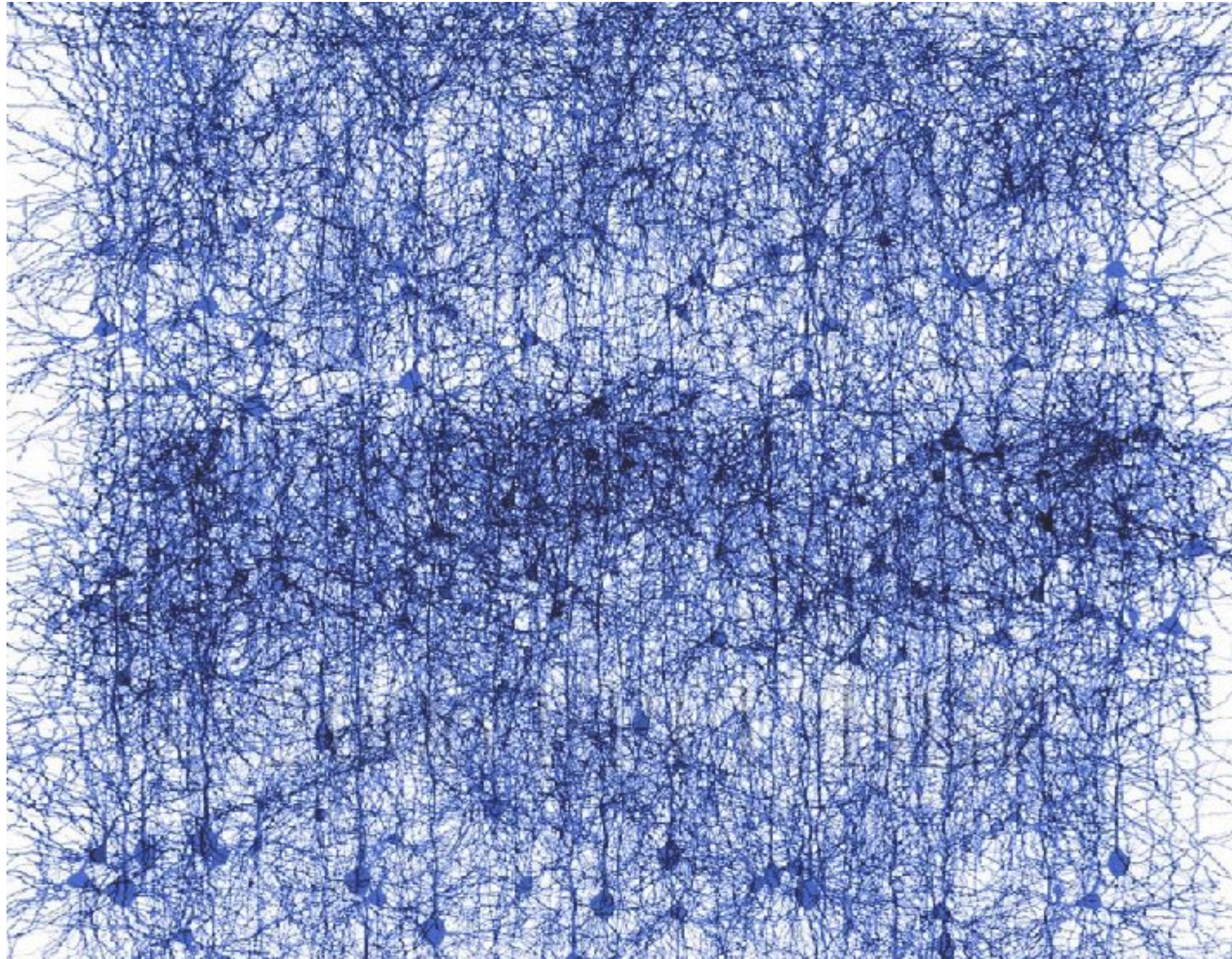
[Hu et al 2013,14,18,20]



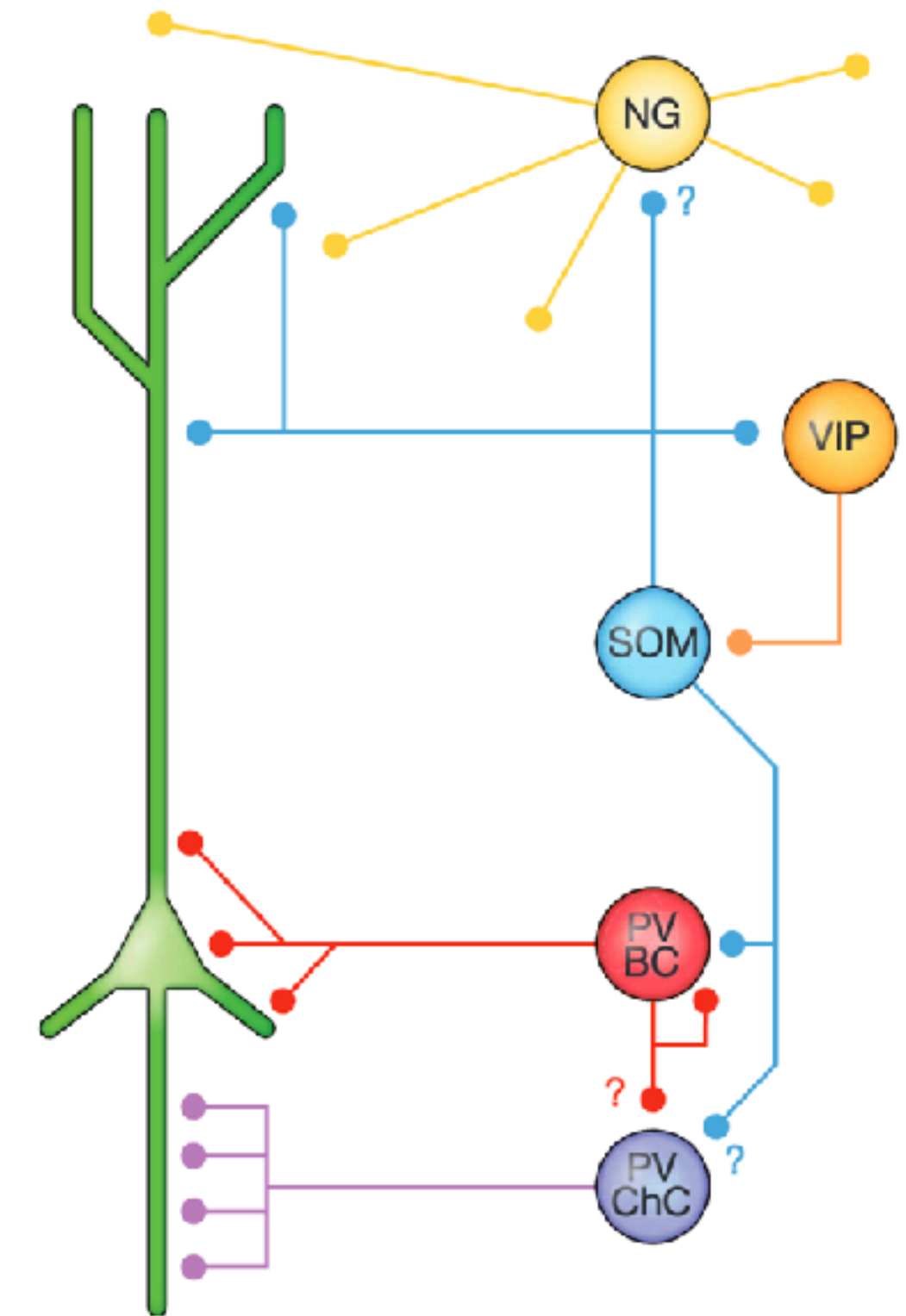
Networks



Networks



Circuits

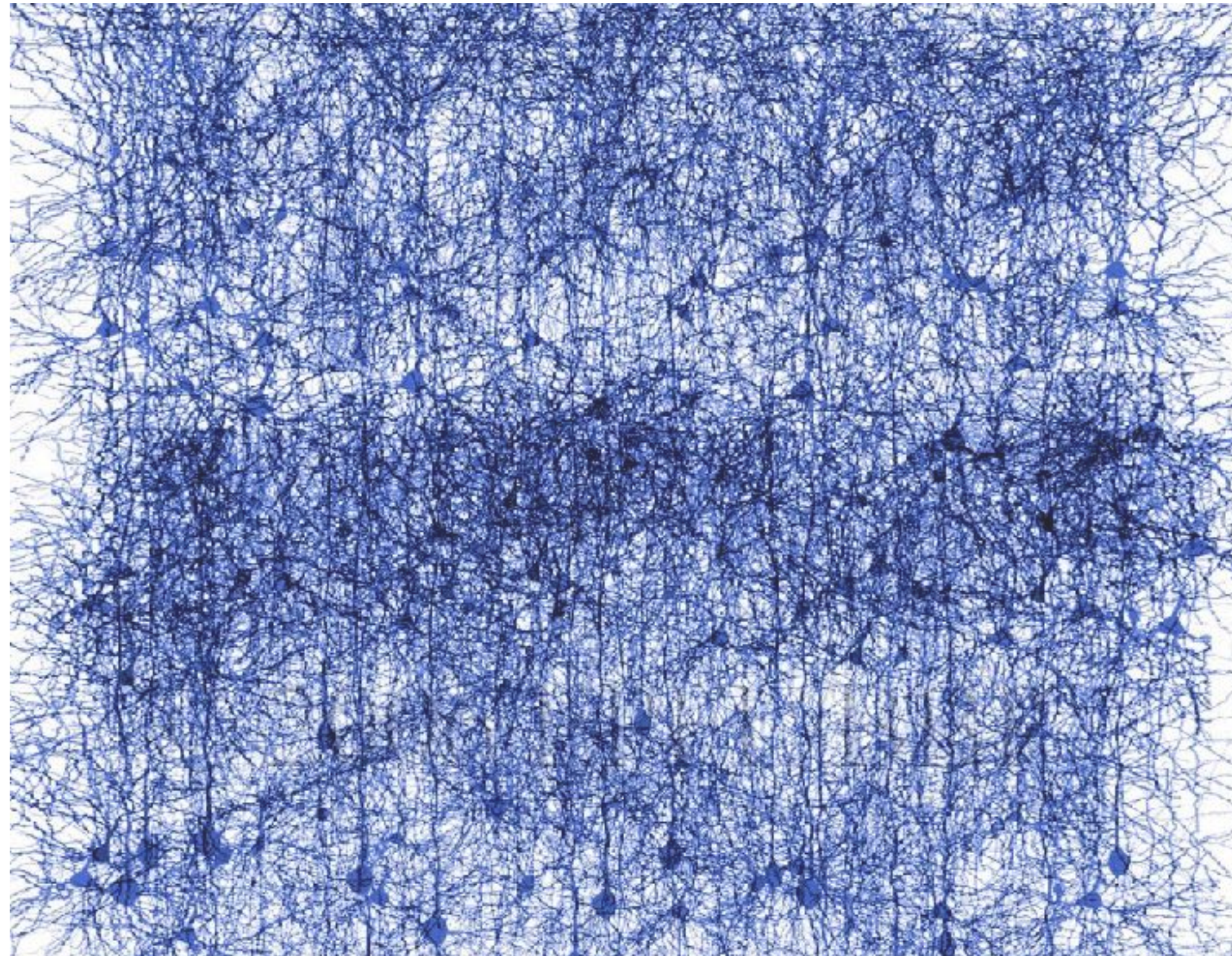


[Harris & Mrsic-Flogel 2013]

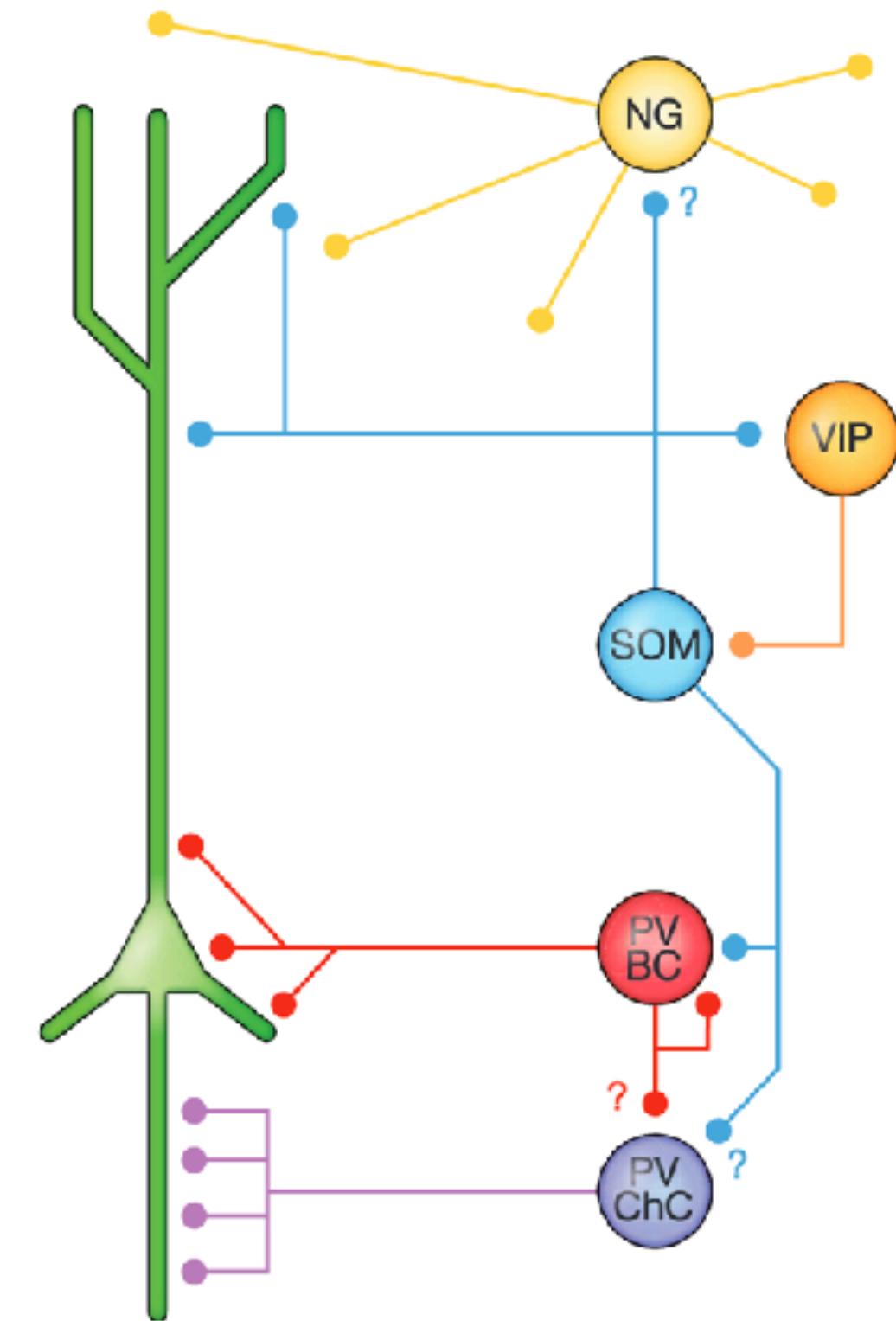
Cell types

- Average activity/inactivations
- Architecture = average connectivity

Networks



Circuits



[Harris & Mrsic-Flogel 2013]

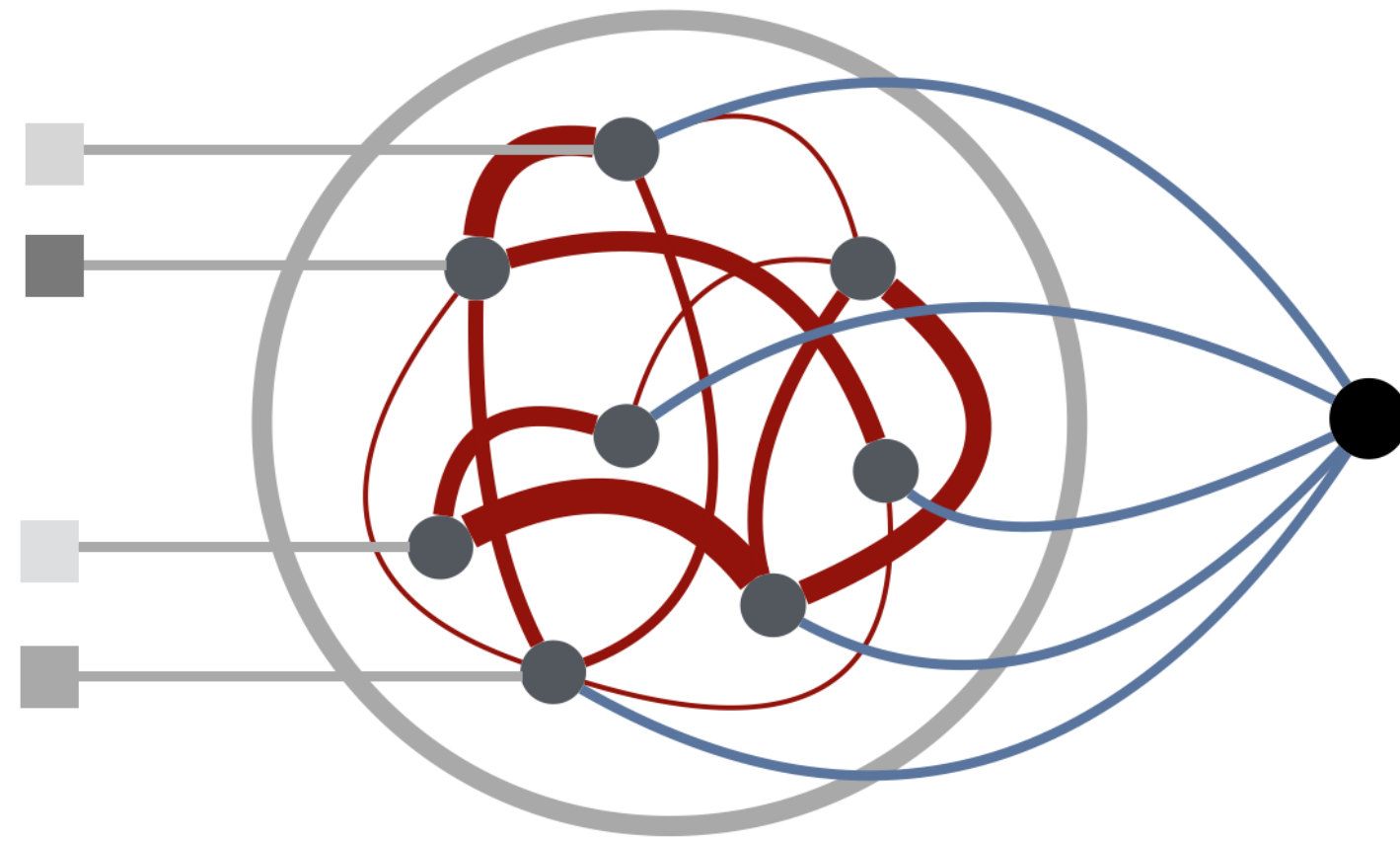
Task variables and computations

- Mixed selectivity & distributed representations
- Low-dimensional dynamics

Cell types

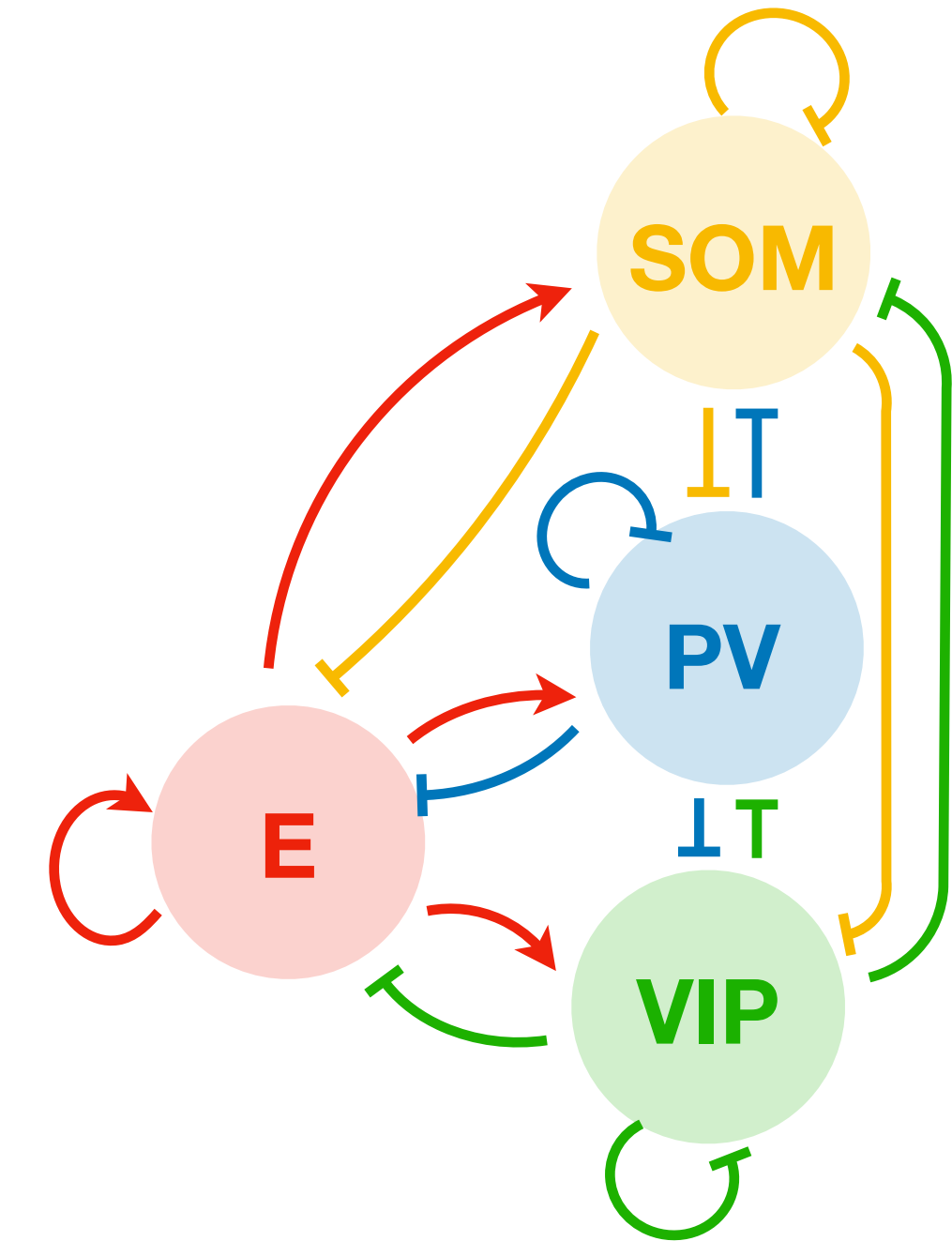
- Average activity/inactivations
- Architecture = average connectivity

Recurrent Neural Networks



[Sussillo 2014, Barak 2017, Yang & Wang 2020]

Circuits



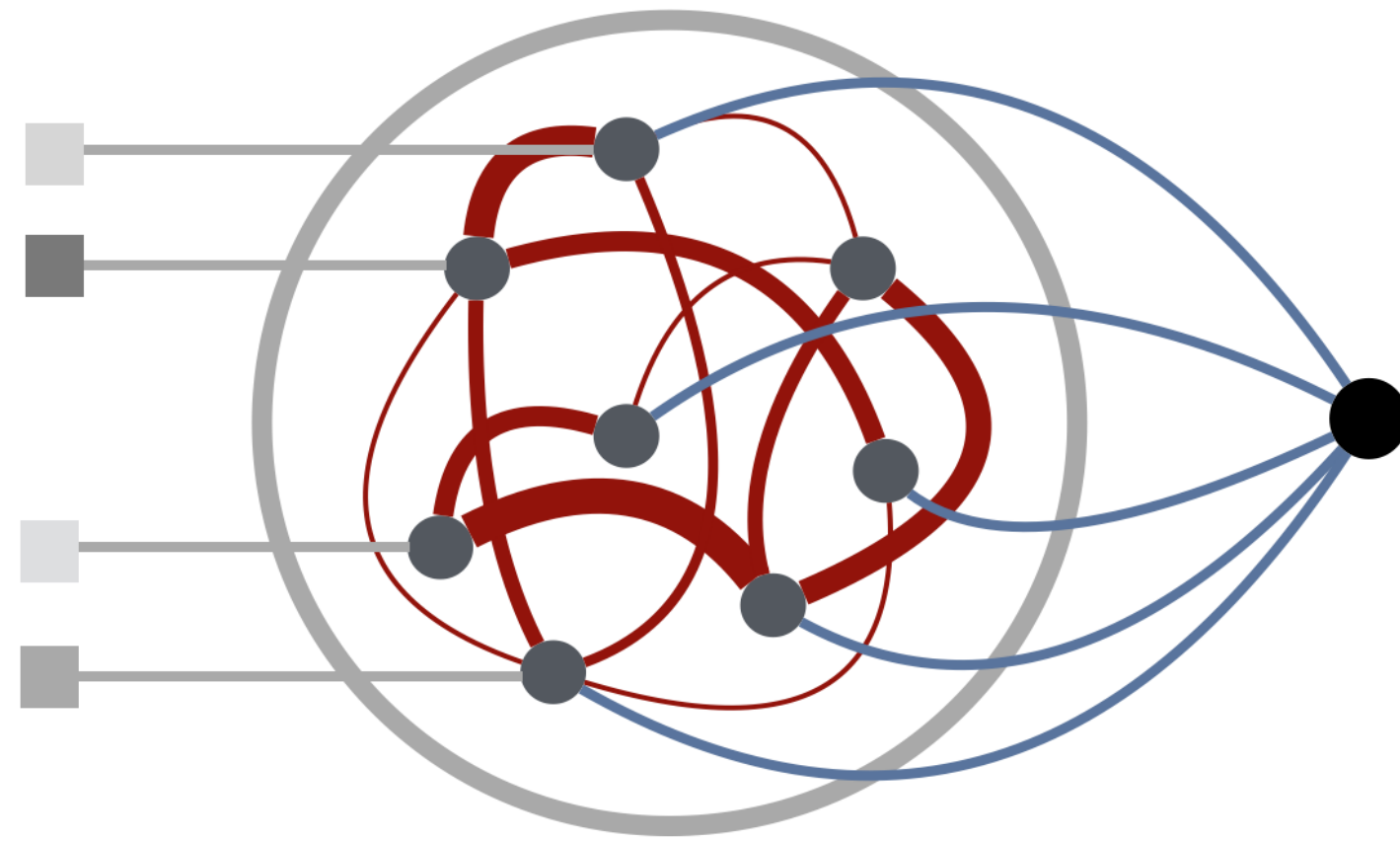
[Wilson and Cowan 1972]

[Litwin-Kumar et al 2016, Palmigiano et al 2020]

Trained networks:

- extract low-d dynamics
- latent variables = abstract computational units

Recurrent Neural Networks

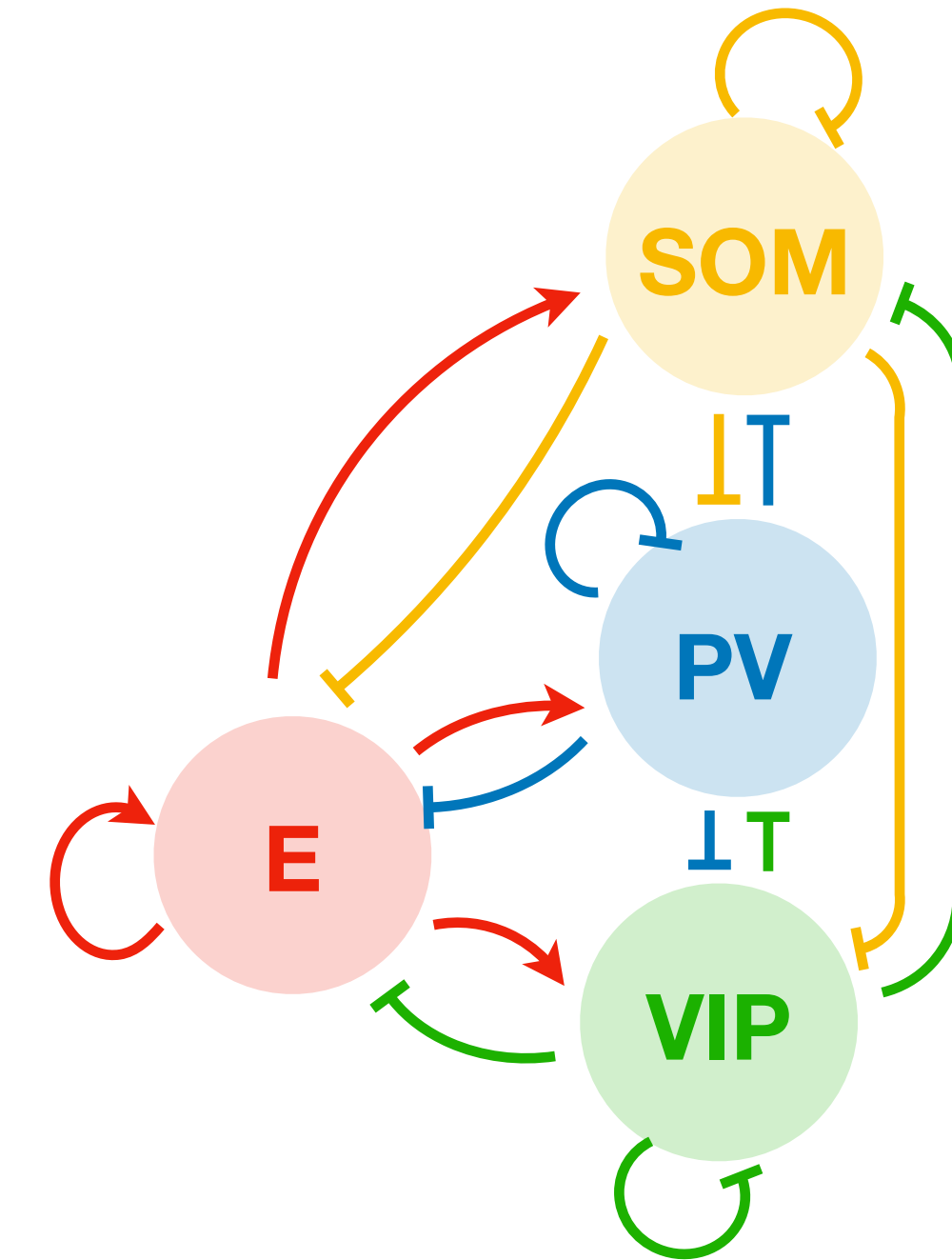


[Sussillo 2014, Barak 2017, Yang & Wang 2020]

Trained networks:

- extract low-d dynamics
- latent variables = abstract computational units

Circuits



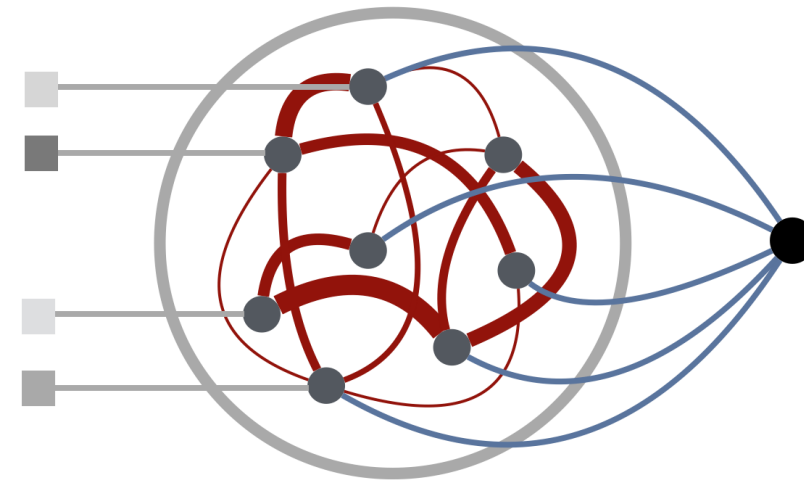
[Wilson and Cowan 1972]

[Litwin-Kumar et al 2016, Palmigiano et al 2020]

Low-dimensional models:

- Units = average cell-type activity
- Connectivity = average cell-type connectivity

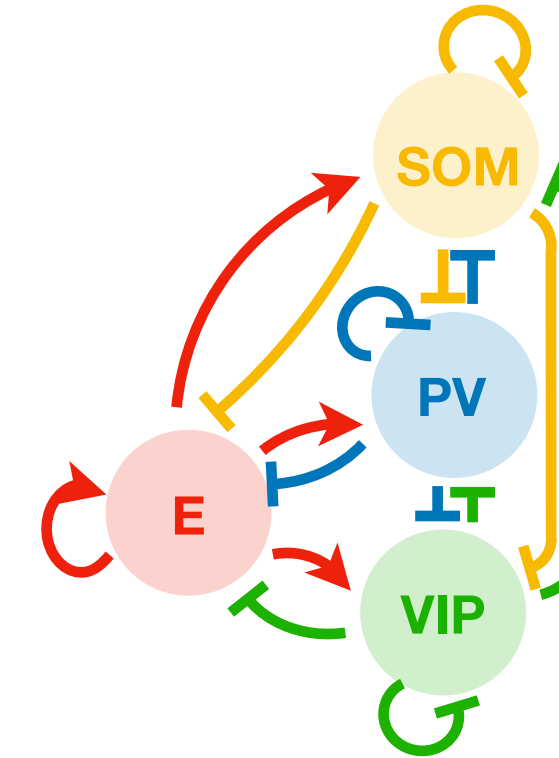
Networks



?

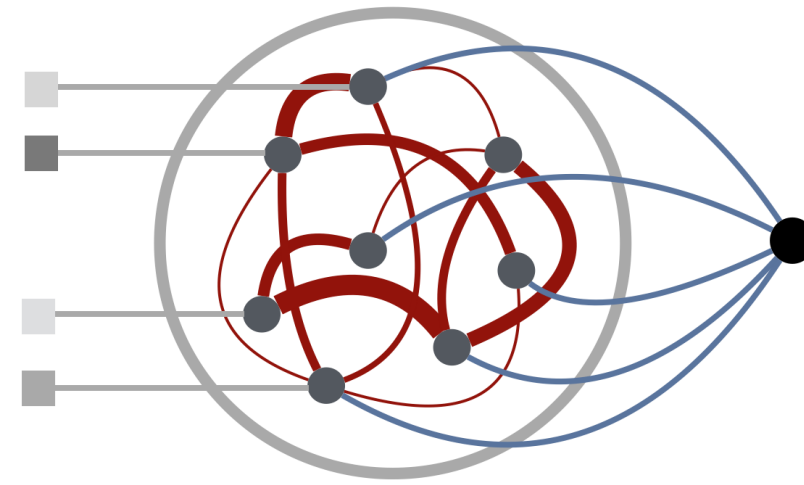


Circuits



1. When/how can we reduce a large network to a low dimensional models?
2. When do low-d units correspond to average cell-type activity?
3. When does average cell-type connectivity correctly predict dynamics?
4. How can the heterogeneity (population) interact with low-d dynamics?

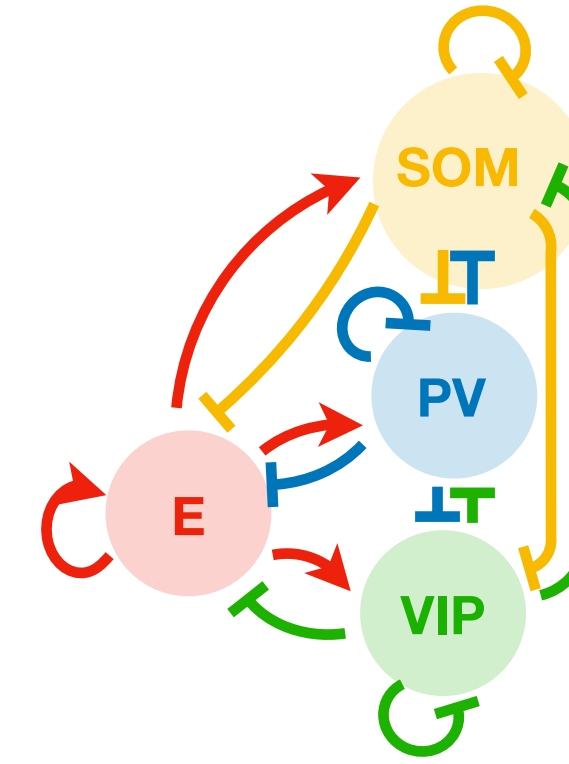
Networks



?



Circuits



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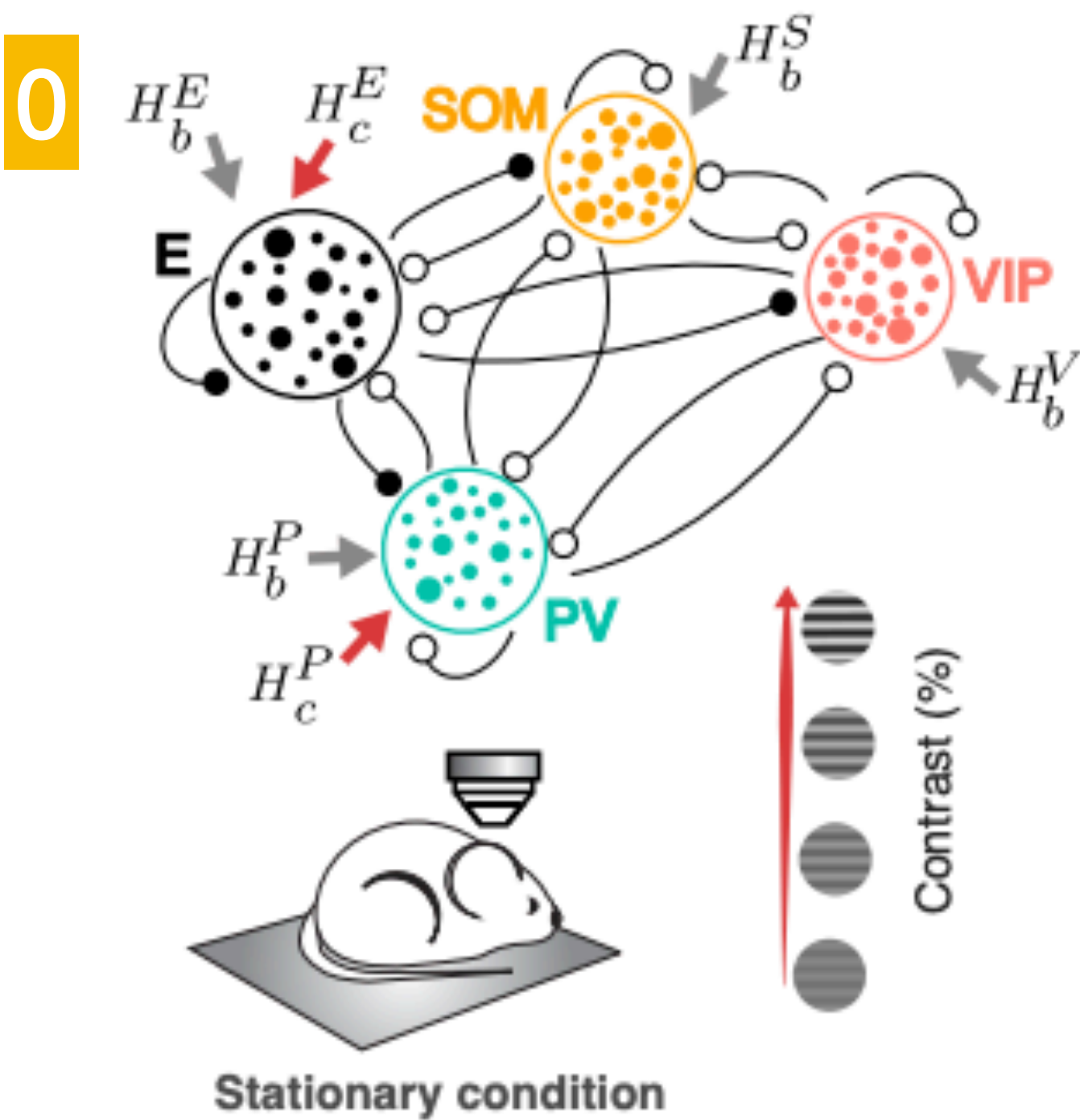
Approaches [Langdon et al 2023]

1. Local connectivity statistics [\[EI networks\]](#)
2. Global connectivity statistics [\[low-rank RNNs\]](#)

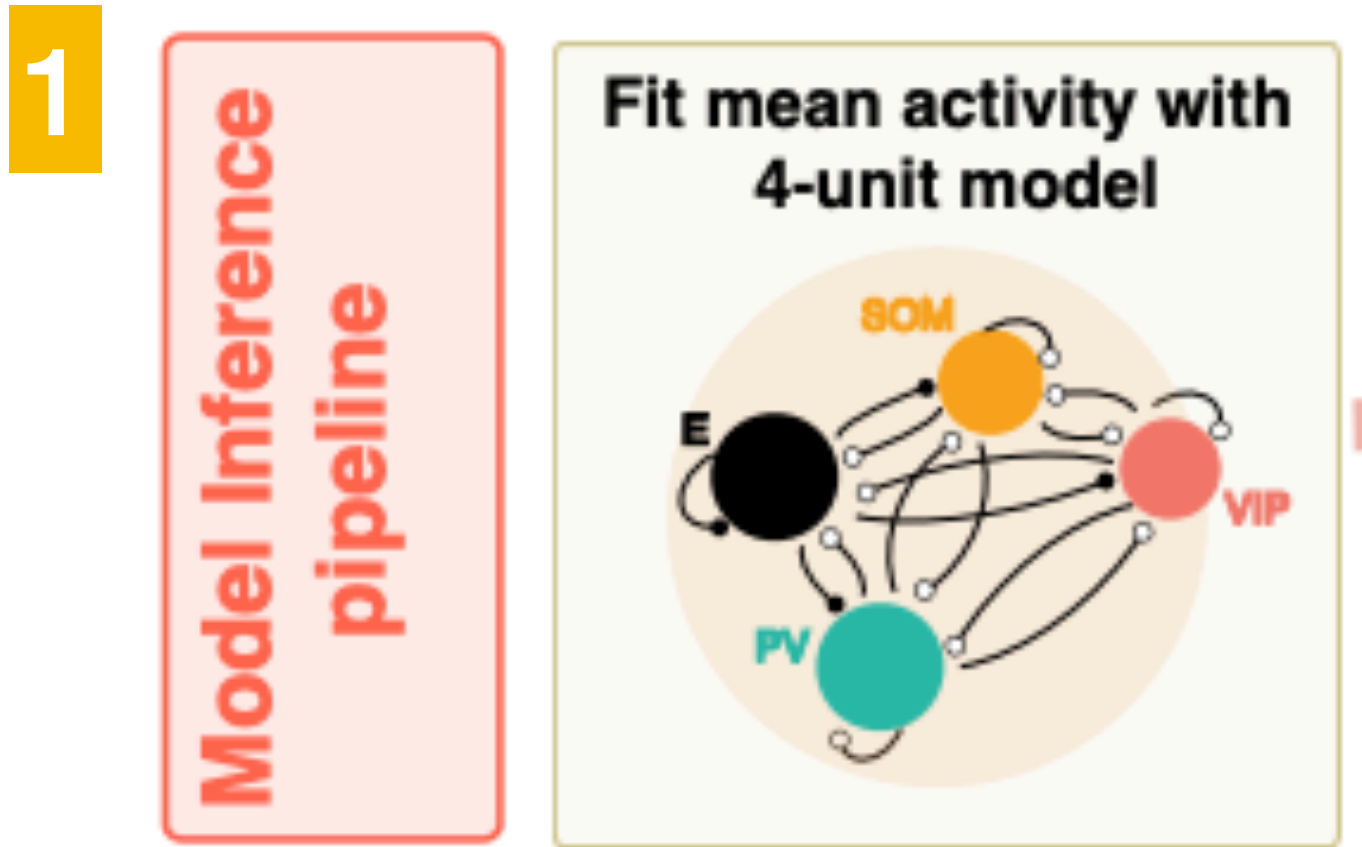


Combine

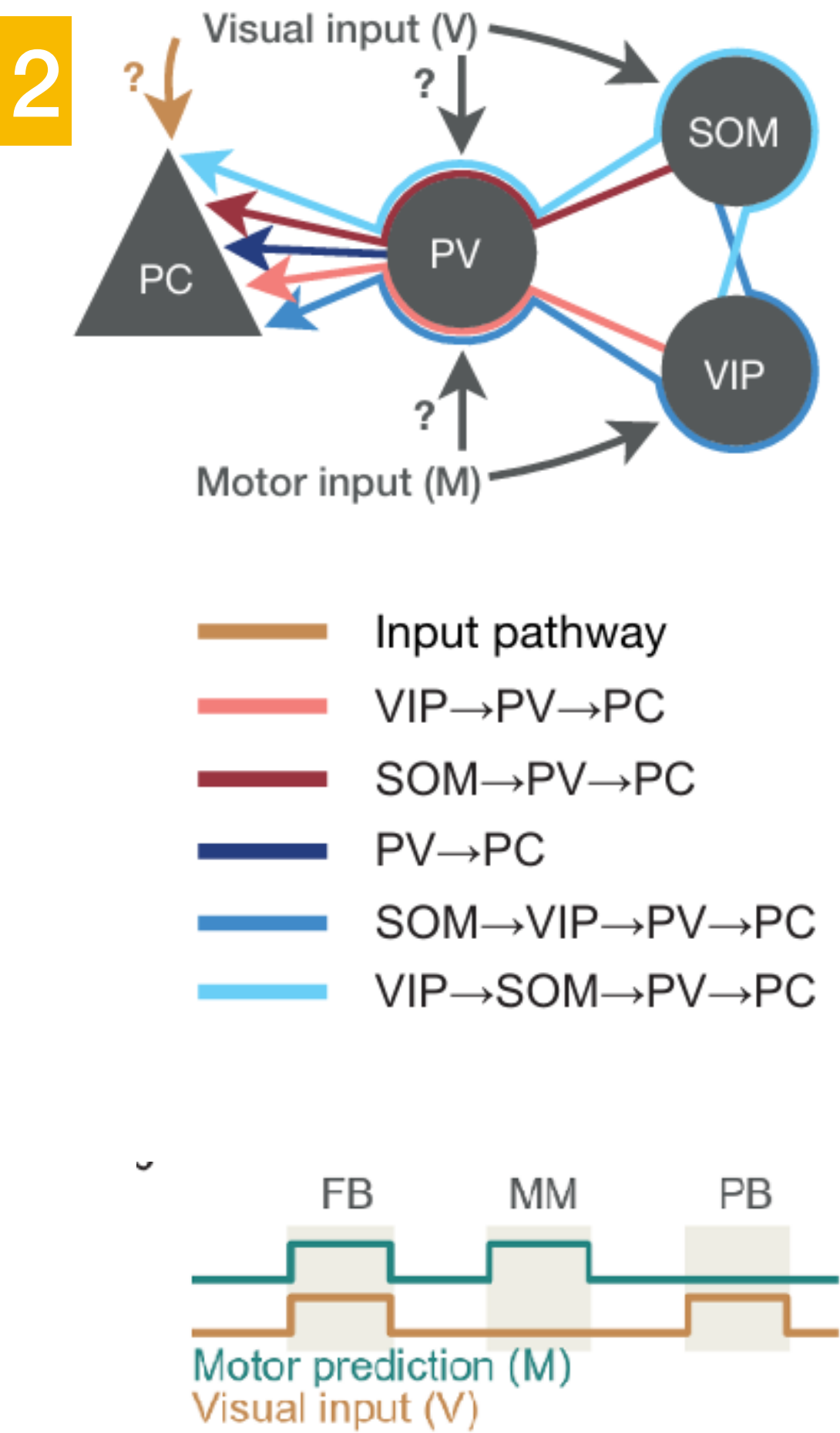
Background: Neural Circuit Model



Optogenetic manipulation for function exploration



Cell-type-specific mean connectivity



Prediction error neurons

Murphy, B. K., & Miller, K. D. 2009,
Palmigiano, et al., 2020,
Hertäg, L., & Sprekeler, H. 2020,
...

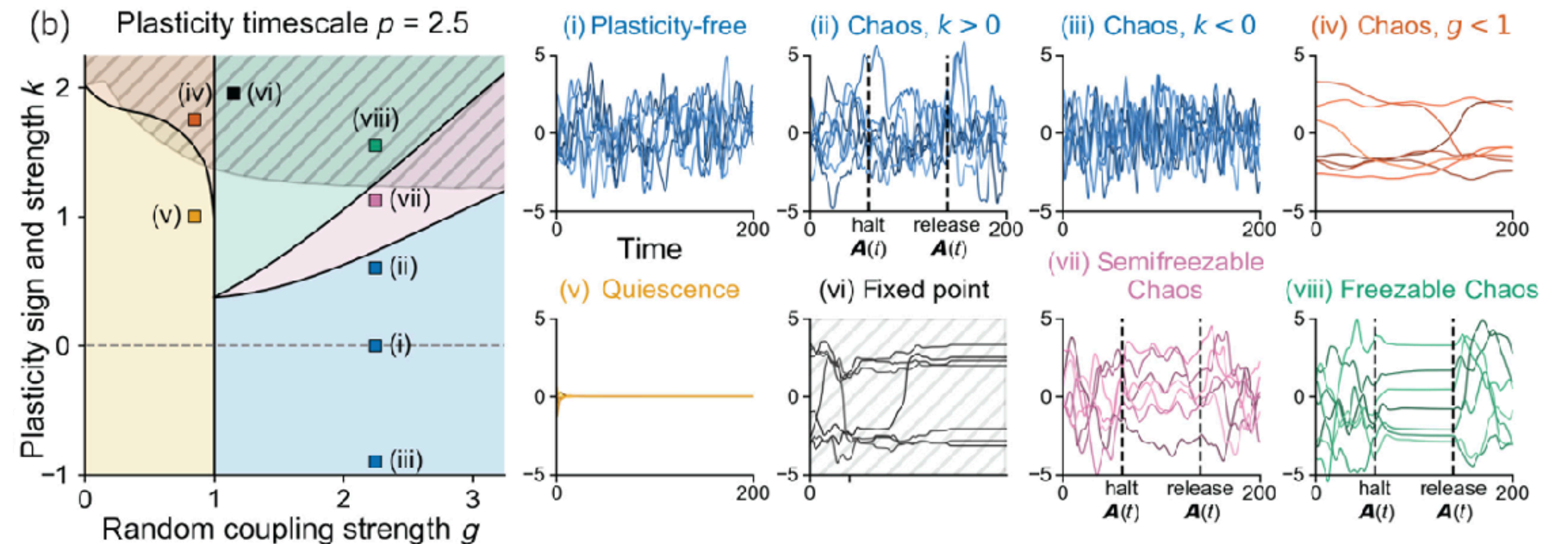
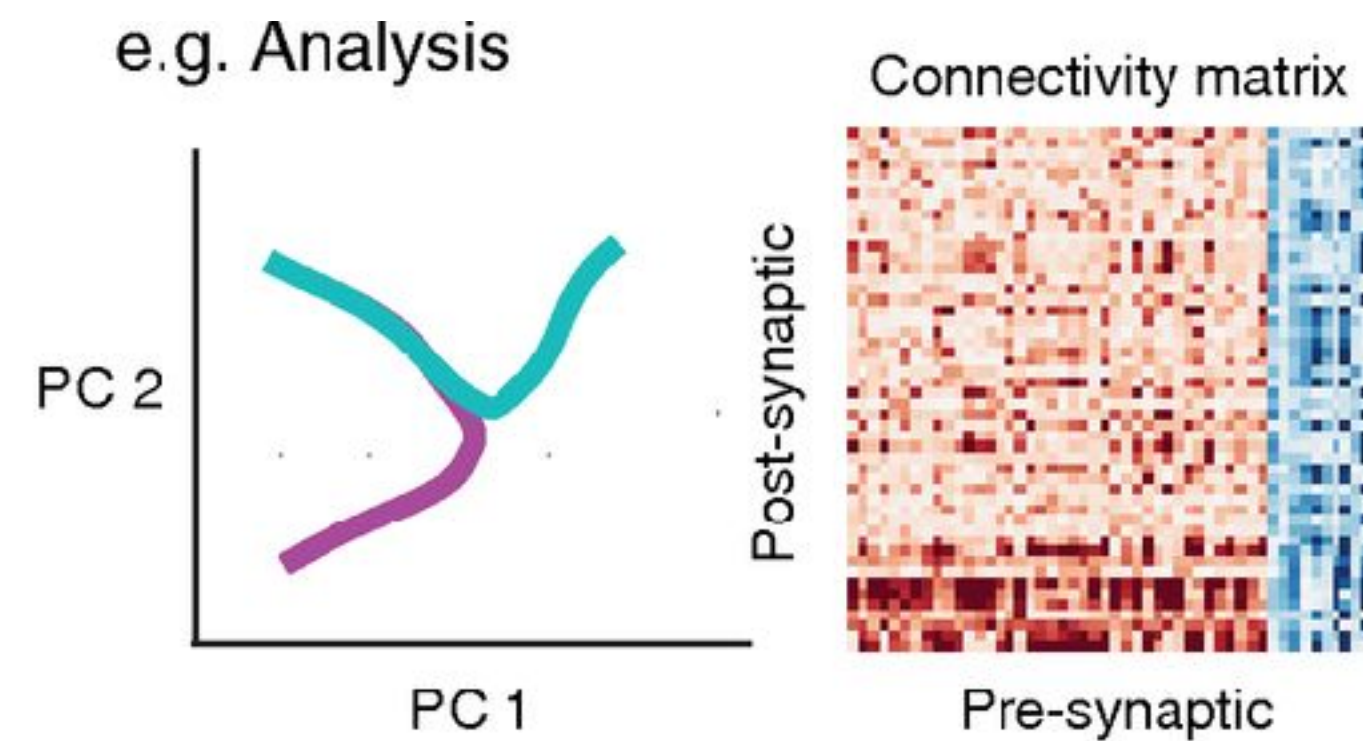
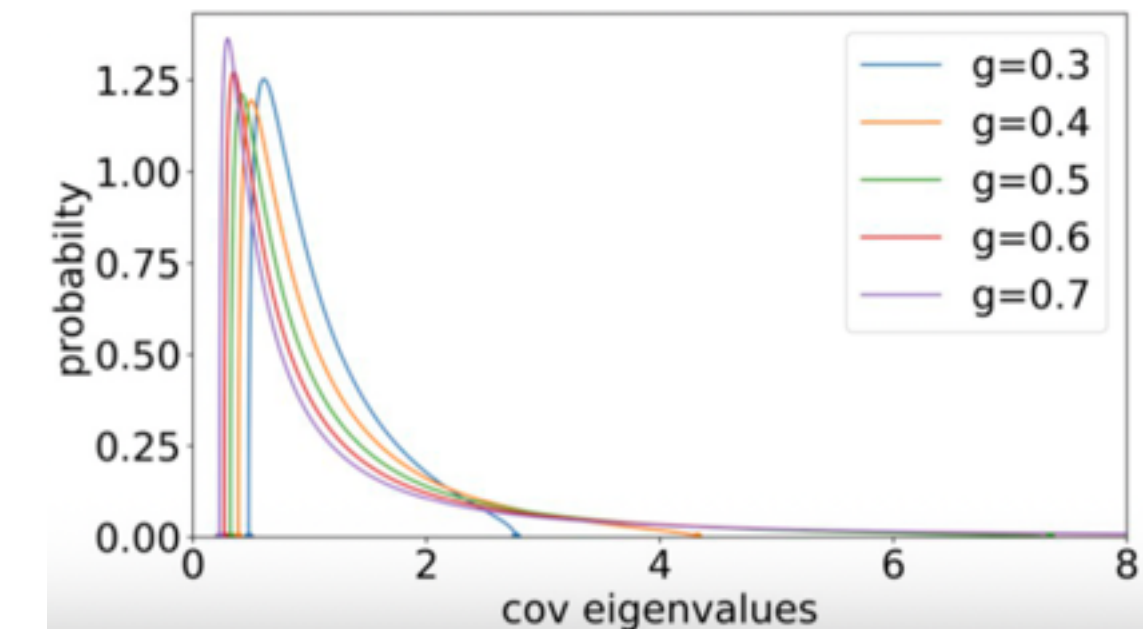
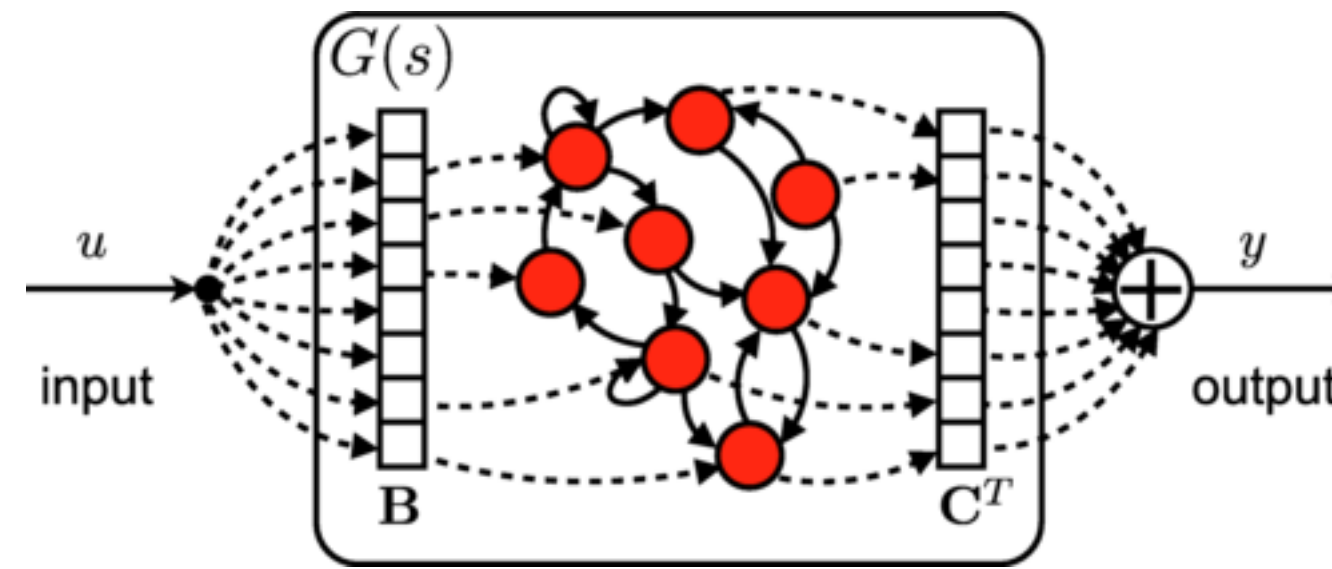
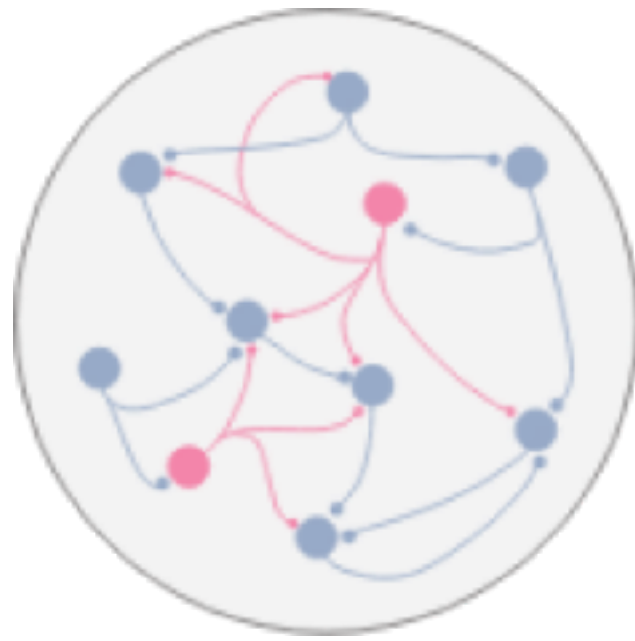
Background: Neural Circuit Model

$$3 \begin{bmatrix} \dot{r}_E \\ \dot{r}_P \\ \dot{r}_S \\ \dot{r}_V \end{bmatrix} = \left(-\mathbf{I} + \begin{bmatrix} w_{EE} & w_{EP} & w_{ES} & w_{EV} \\ w_{PE} & w_{PP} & w_{PS} & w_{PV} \\ w_{SE} & w_{SP} & w_{SS} & w_{SV} \\ w_{VE} & w_{VP} & w_{VS} & w_{VV} \end{bmatrix} \right) \begin{bmatrix} r_E \\ r_P \\ r_S \\ r_V \end{bmatrix}$$

Characteristics and Limitations of Classic Neural Circuit Models

- High-dimensional dynamics degrade to low-dimensional representations
- Individual neurons assumed to have identical activity within populations
- Higher-order connectivity and structured (fluctuated) neural activity patterns ignored
- Advanced version has the reliance on ad-hoc noise models rather than principled variability

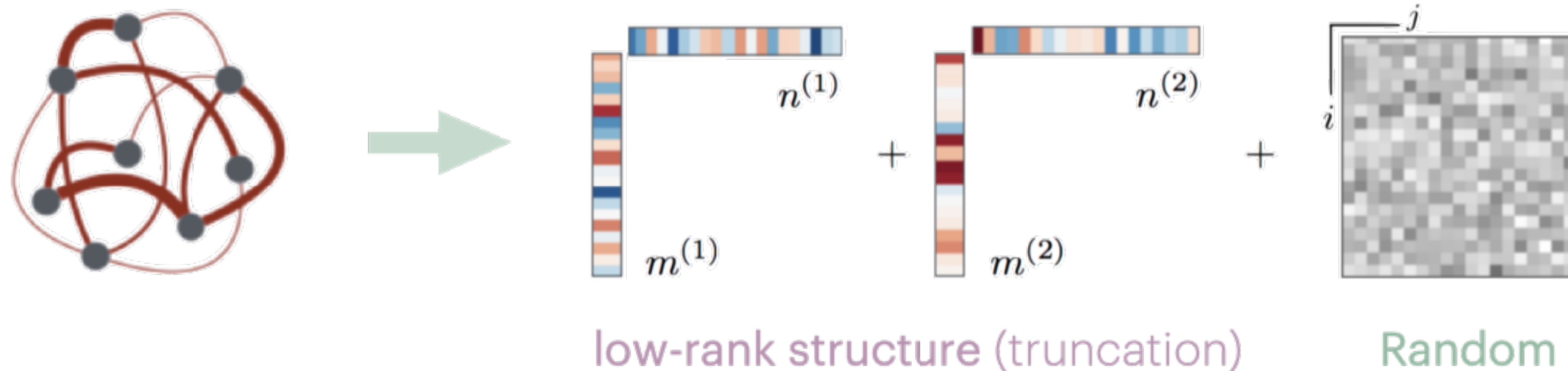
Background: Random Recurrent Neural Network



Sompolinsky, Cristanti & Sommer, 1988
 Ranjan, Abbott & Sompolinsky, 2010
 Hu, et al., 2013, 2018, 2020
 Ehrlich et al, 2020
 Clark, Abbott, & Litwin-Kumar, 2023
 Clark & Abbott, 2024

Background: Random Recurrent Neural Network

— Low-rank Recurrent Neural Networks (lrRNNs)



The diagram illustrates the decomposition of a recurrent neural network into low-rank and random components. On the left, a network graph with nodes and edges is shown. To the right, the weight matrix J_{ij} is decomposed into two parts: a low-rank structure (truncation) and a random component. The low-rank structure is represented by the sum of products of matrices $m^{(1)}$, $n^{(1)}$, $m^{(2)}$, $n^{(2)}$, ..., $m^{(r)}$, $n^{(r)}$. The random component is represented by the sum of products of matrices $m^{(r+1)}$, $n^{(r+1)}$, ..., $m^{(N)}$, $n^{(N)}$.

$$J_{ij} = m_i^{(1)} n_j^{(1)} + m_i^{(2)} n_j^{(2)} + \dots + m_i^{(r)} n_j^{(r)} + m_i^{(r+1)} n_j^{(r+1)} + \dots + m_i^{(N)} n_j^{(N)}$$

low-rank structure (truncation) Random component

$$\frac{d}{dt} K_1 = -K_1 + \tilde{\sigma}_{n_1 m_1} K_1 + \tilde{\sigma}_{n_1 I} V$$

$$\tilde{\sigma} = \langle \phi' \rangle \sigma \quad \text{Gain modulation}$$

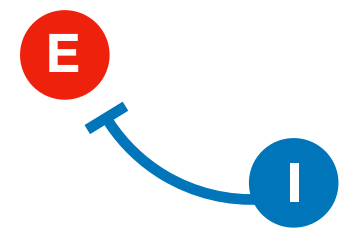
$$\tilde{\sigma} = \langle \phi' \rangle \times$$



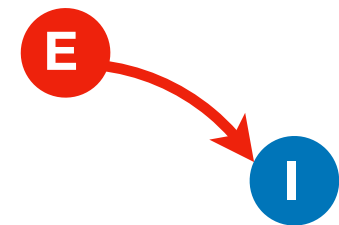
Mastrogiuseppe & Ostojic 2018
 Beiran et al, Neural Comp 2021
 Dubreuil*, Valente* et al, Nat Neuro 2022

Local Connectivity: EI networks

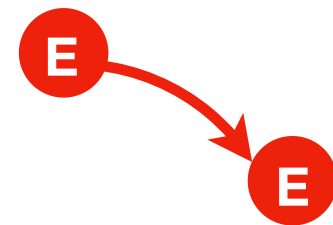
Statistics of connection strengths



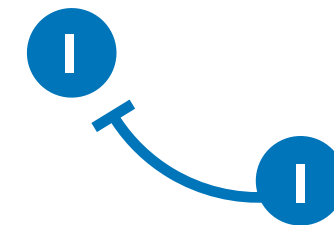
$Prob(J_{EI})$



$Prob(J_{IE})$

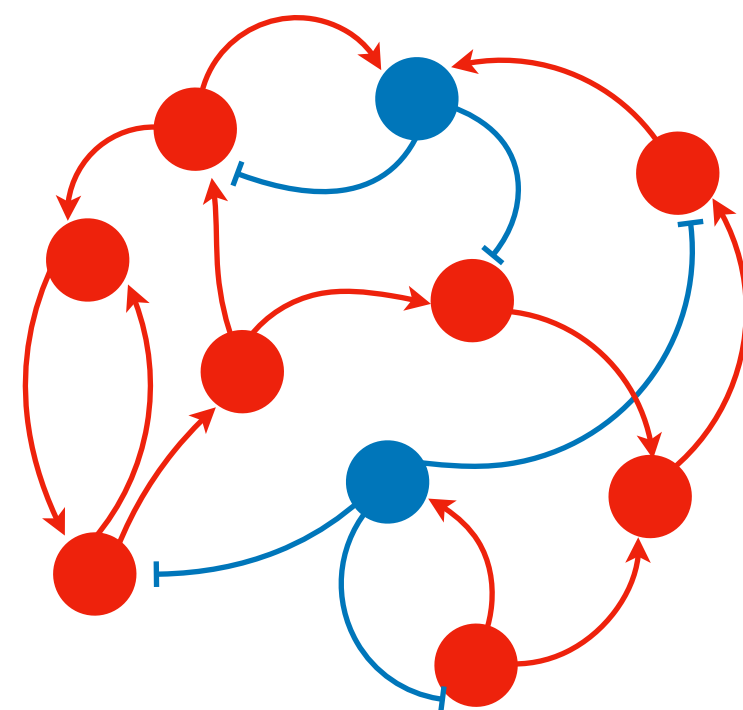


$Prob(J_{EE})$



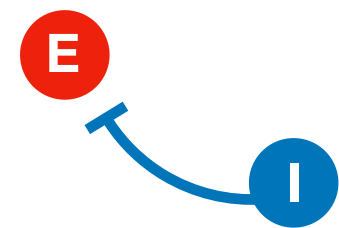
$Prob(J_{II})$

Excitatory-Inhibitory Network (4:1)

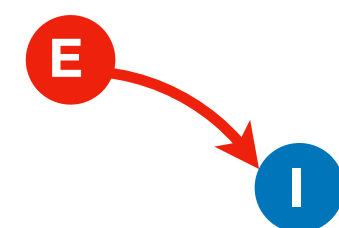


Local Connectivity: EI networks

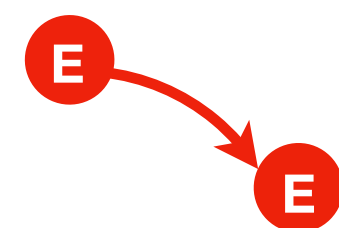
Statistics of connection strengths



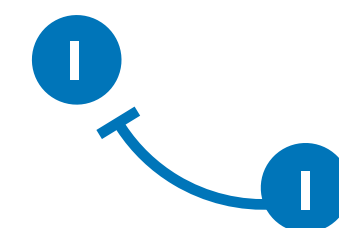
$Prob(J_{EI})$



$Prob(J_{IE})$

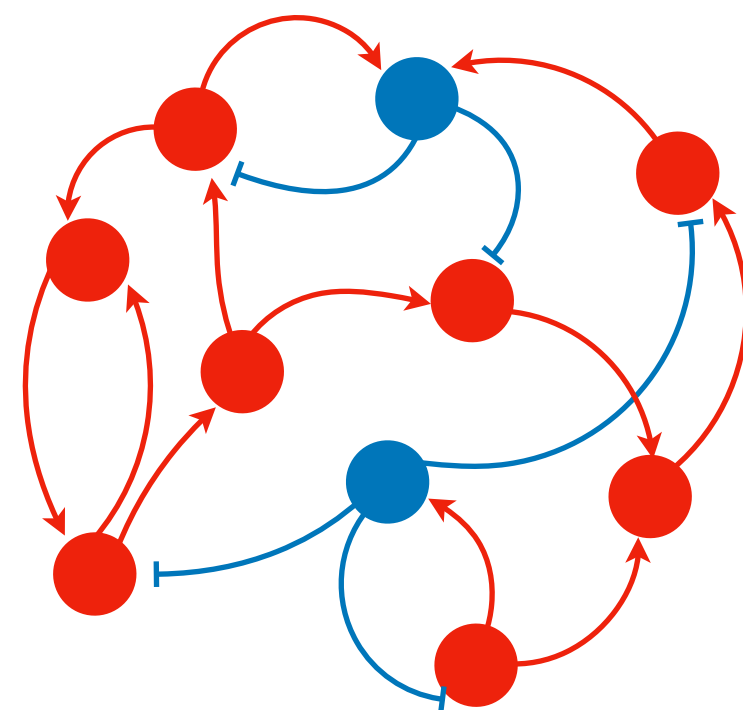


$Prob(J_{EE})$



$Prob(J_{II})$

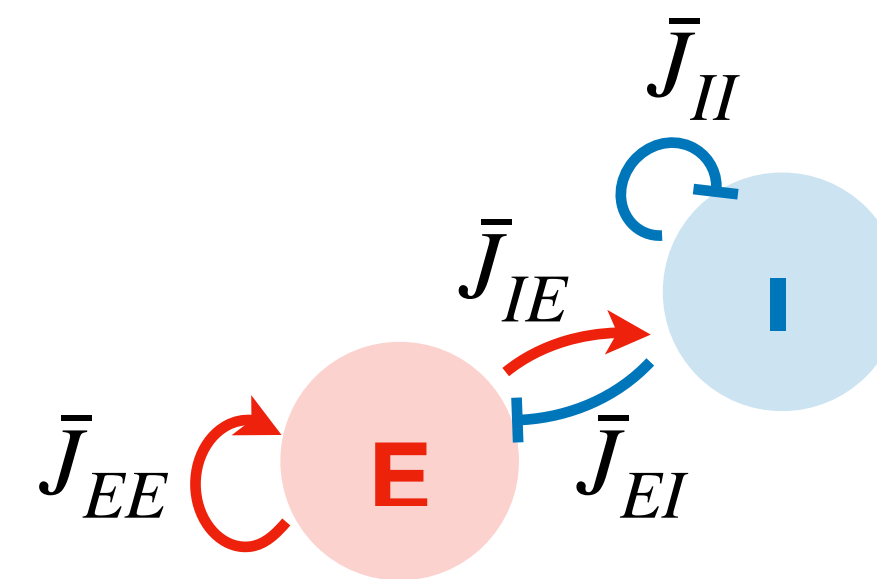
Excitatory-Inhibitory Network



Mean-field theory

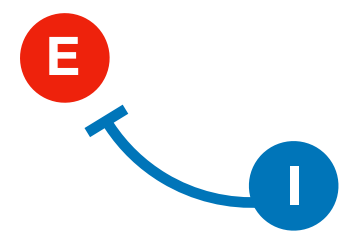


Effective EI Circuit

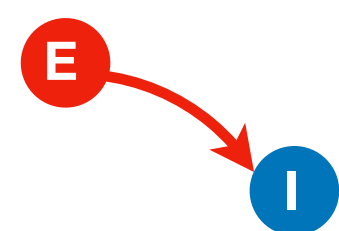


Local Connectivity: EI networks

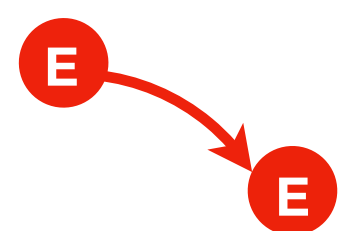
Statistics of connection strengths



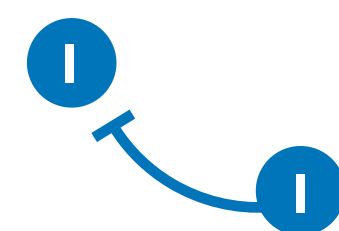
$Prob(J_{EI})$



$Prob(J_{IE})$

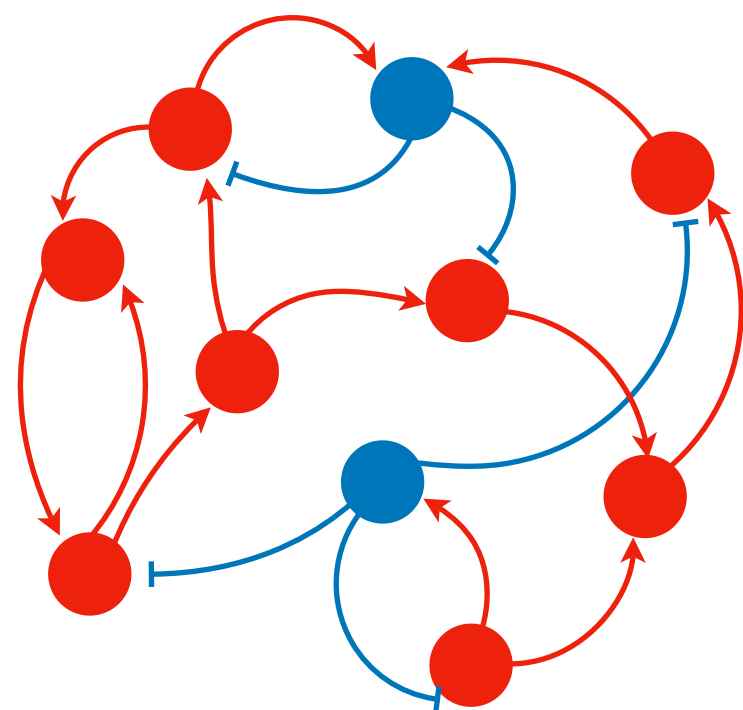


$Prob(J_{EE})$



$Prob(J_{II})$

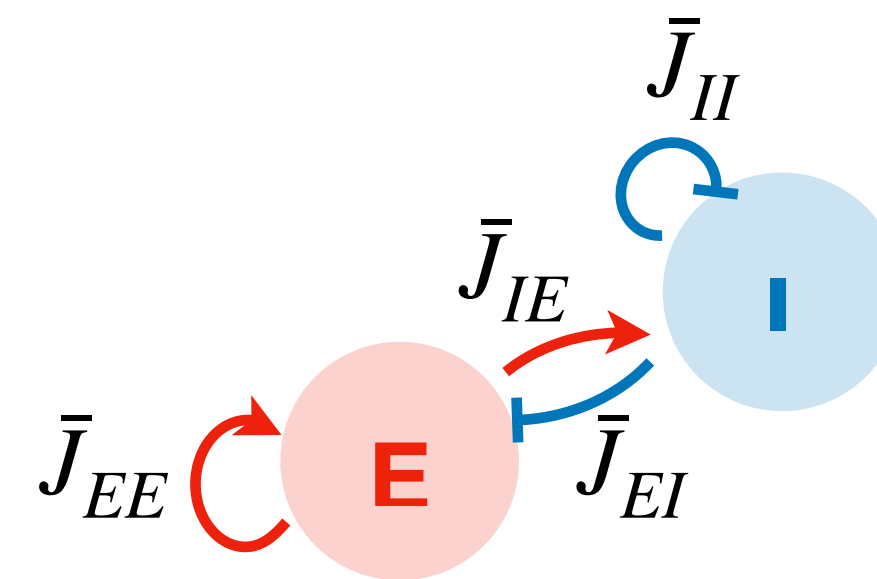
Excitatory-Inhibitory Network



Mean-field theory



Effective EI Circuit



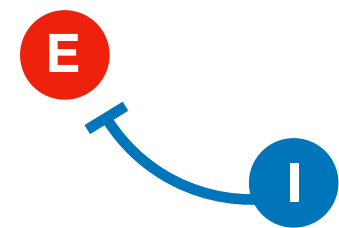
Low-dimensional model:

- Unit = average cell-type activity
- Connectivity = average cell-type connectivity

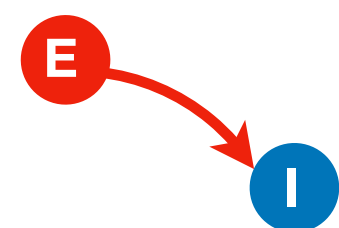
[van Vreeswijk & Sompolinsky 1996,1998; Amit & Brunel 1997]

Local Connectivity: EI networks

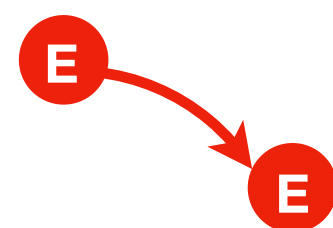
Statistics of connection strengths



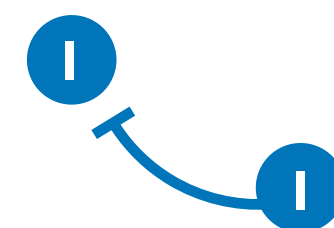
$Prob(J_{EI})$



$Prob(J_{IE})$



$Prob(J_{EE})$

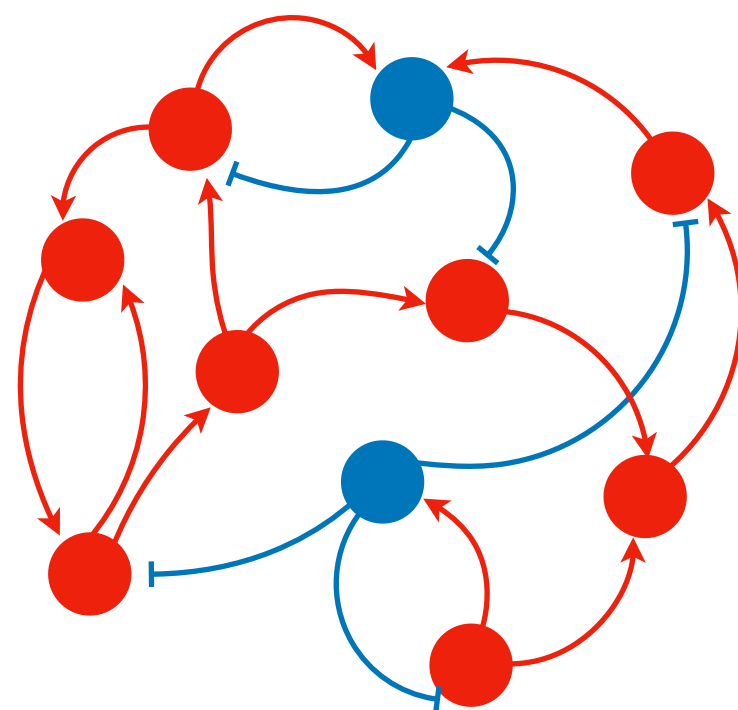


$Prob(J_{II})$

Assumptions:

- Cell-type dependence
- No correlations

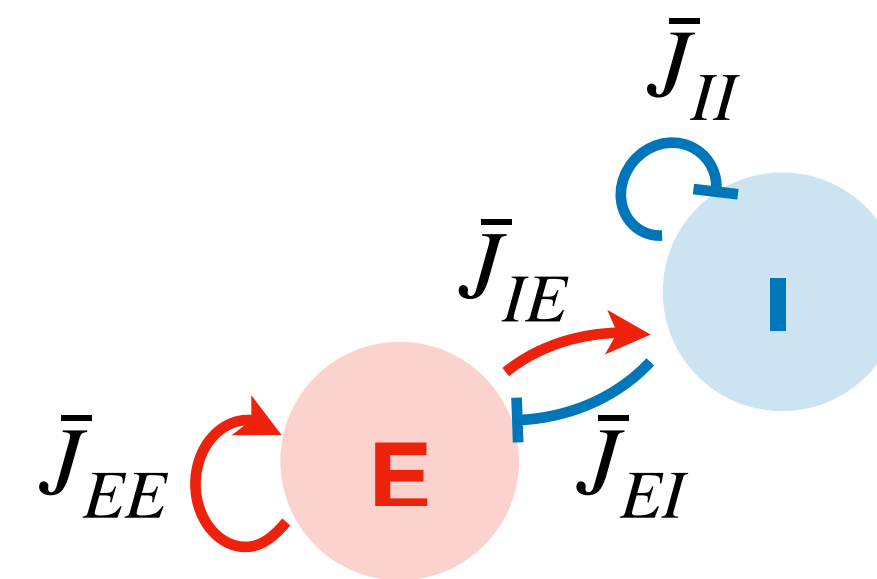
Excitatory-Inhibitory Network



Mean-field theory



Effective EI Circuit



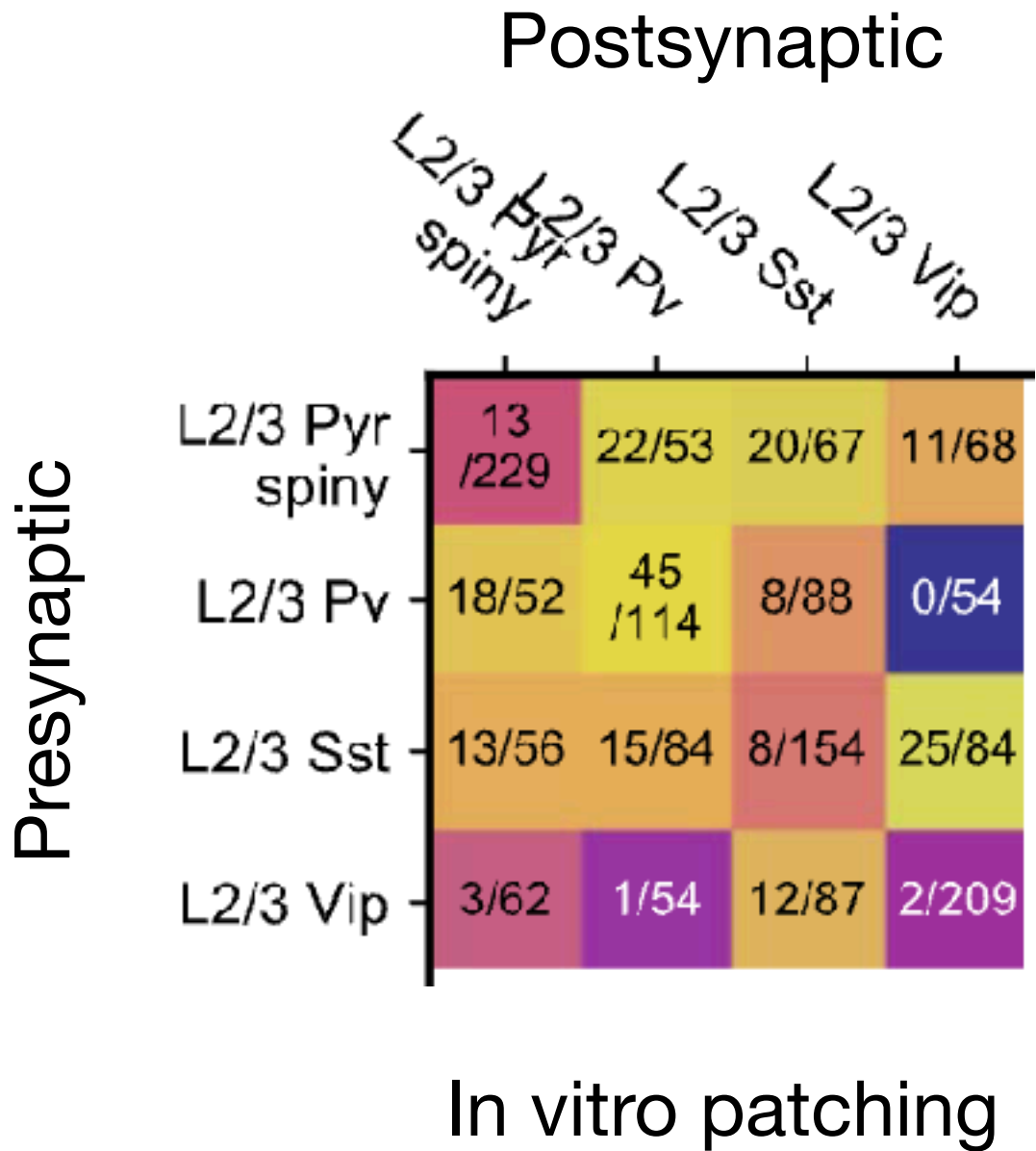
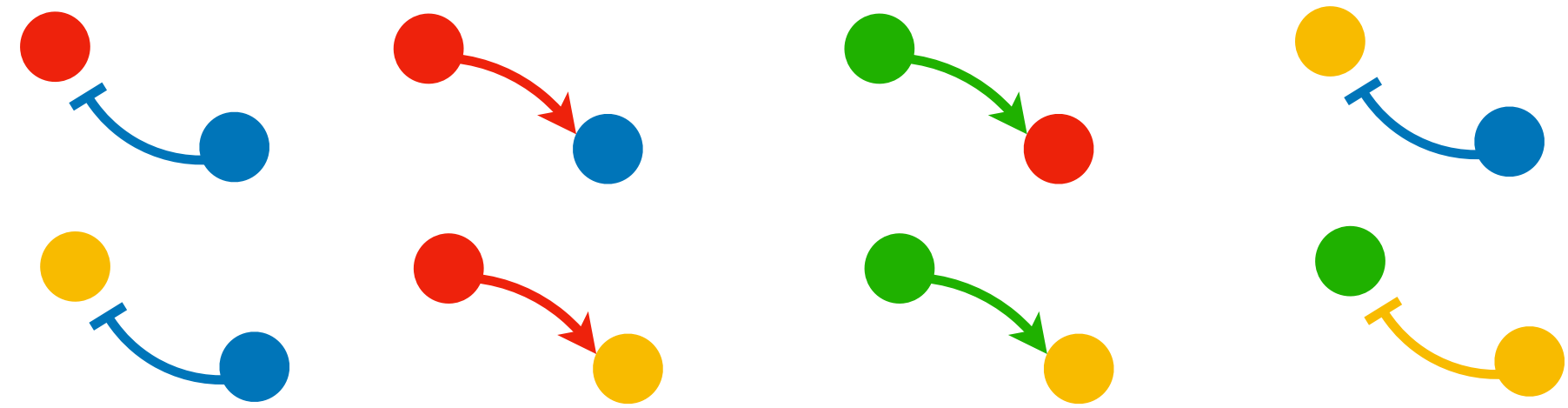
Low-dimensional model:

- Unit = average cell-type activity
- Connectivity = average cell-type connectivity

[van Vreeswijk & Sompolinsky 1996,1998; Amit & Brunel 1997]

Local Connectivity

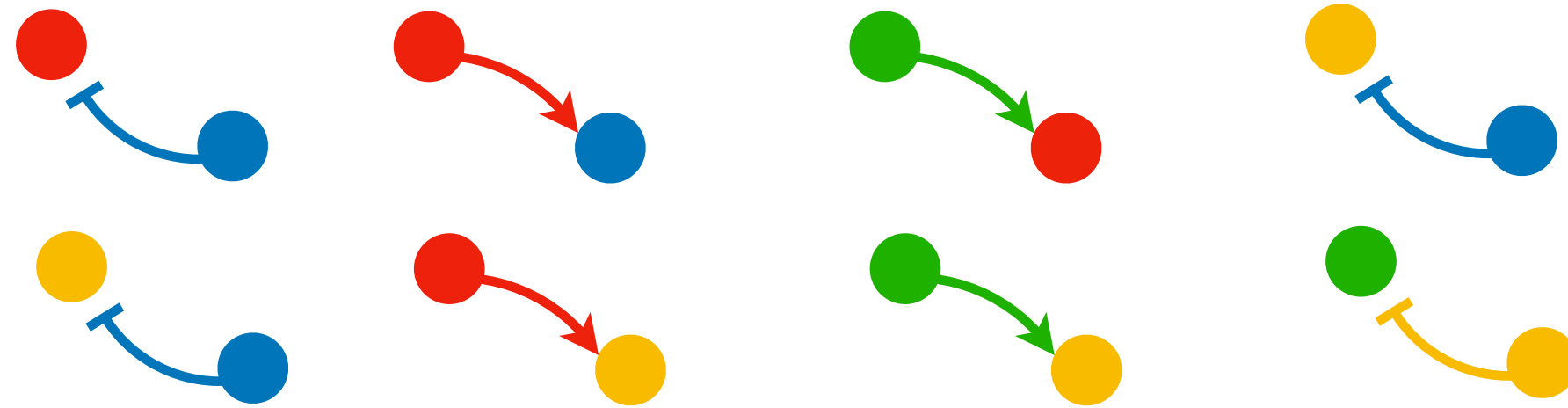
Connection probabilities



[Campagnola et al 2022]

Local Connectivity

Connection probabilities



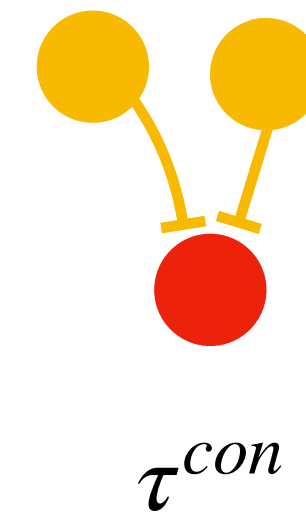
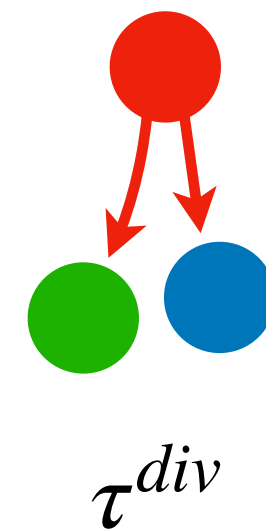
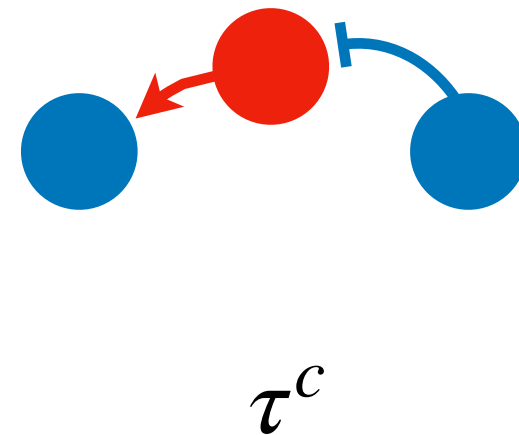
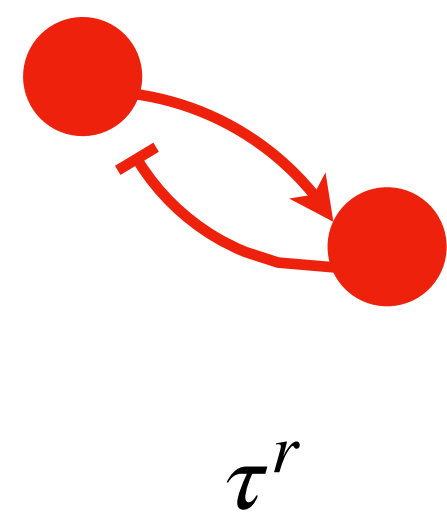
Correlations = *Connectivity motifs*

reciprocal

chain

divergent

convergent

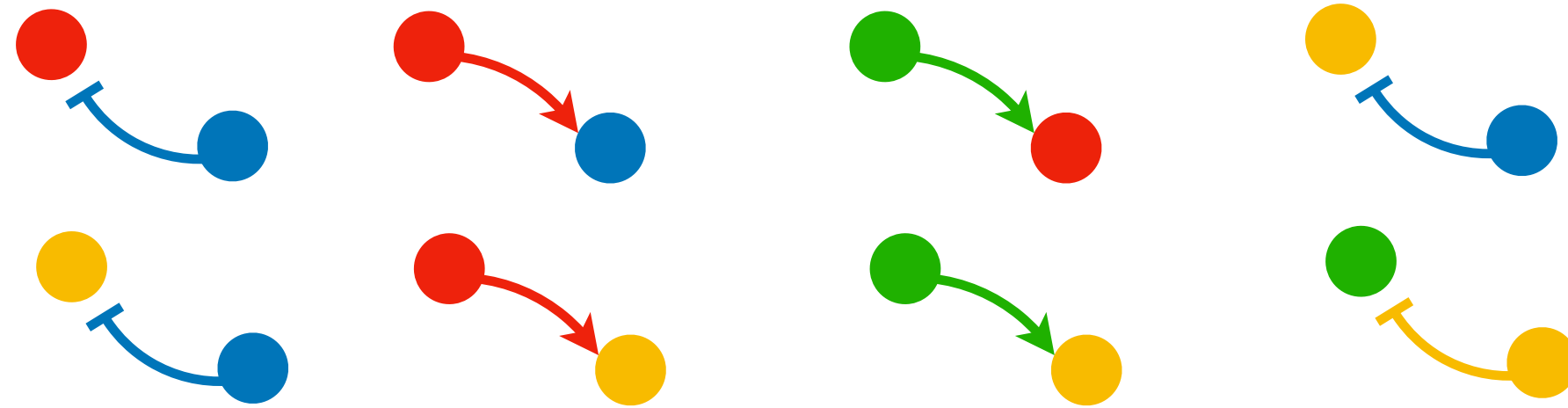


		Postsynaptic			
		L2/3 Pyr spiny	L2/3 Pyr	L2/3 Sst	L2/3 Vip
Presynaptic	L2/3 Pyr spiny	13/229	22/53	20/67	11/68
	L2/3 Pv	18/52	45/114	8/88	0/54
	L2/3 Sst	13/56	15/84	8/154	25/84
	L2/3 Vip	3/62	1/54	12/87	2/209

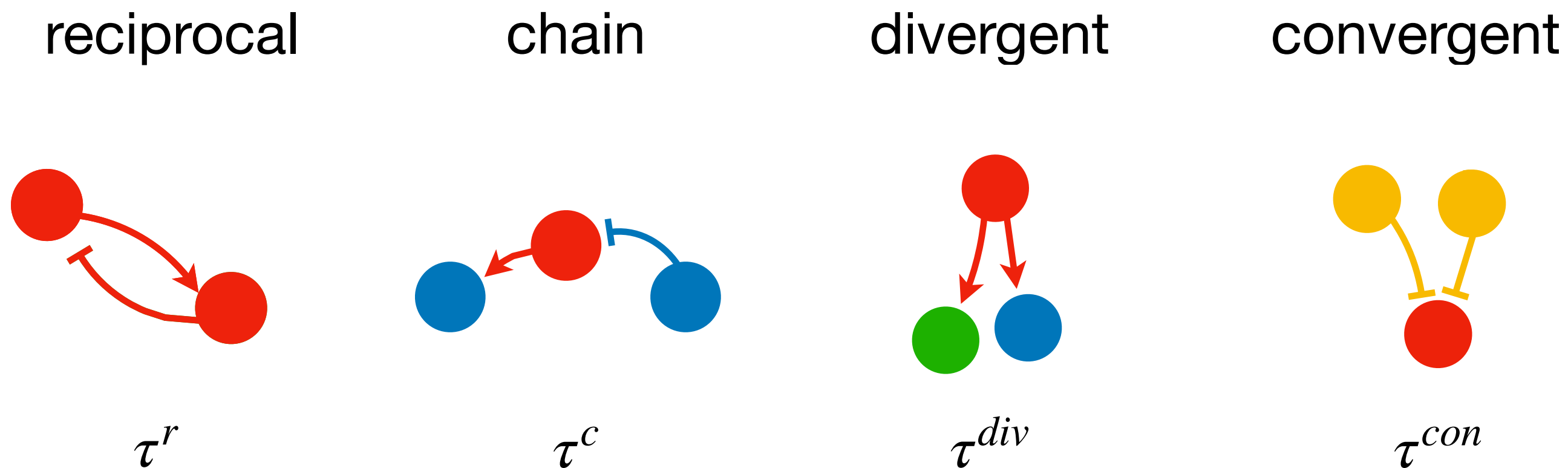
[Campagnola et al 2022]

Local Connectivity

Connection probabilities



Correlations = *Connectivity motifs*



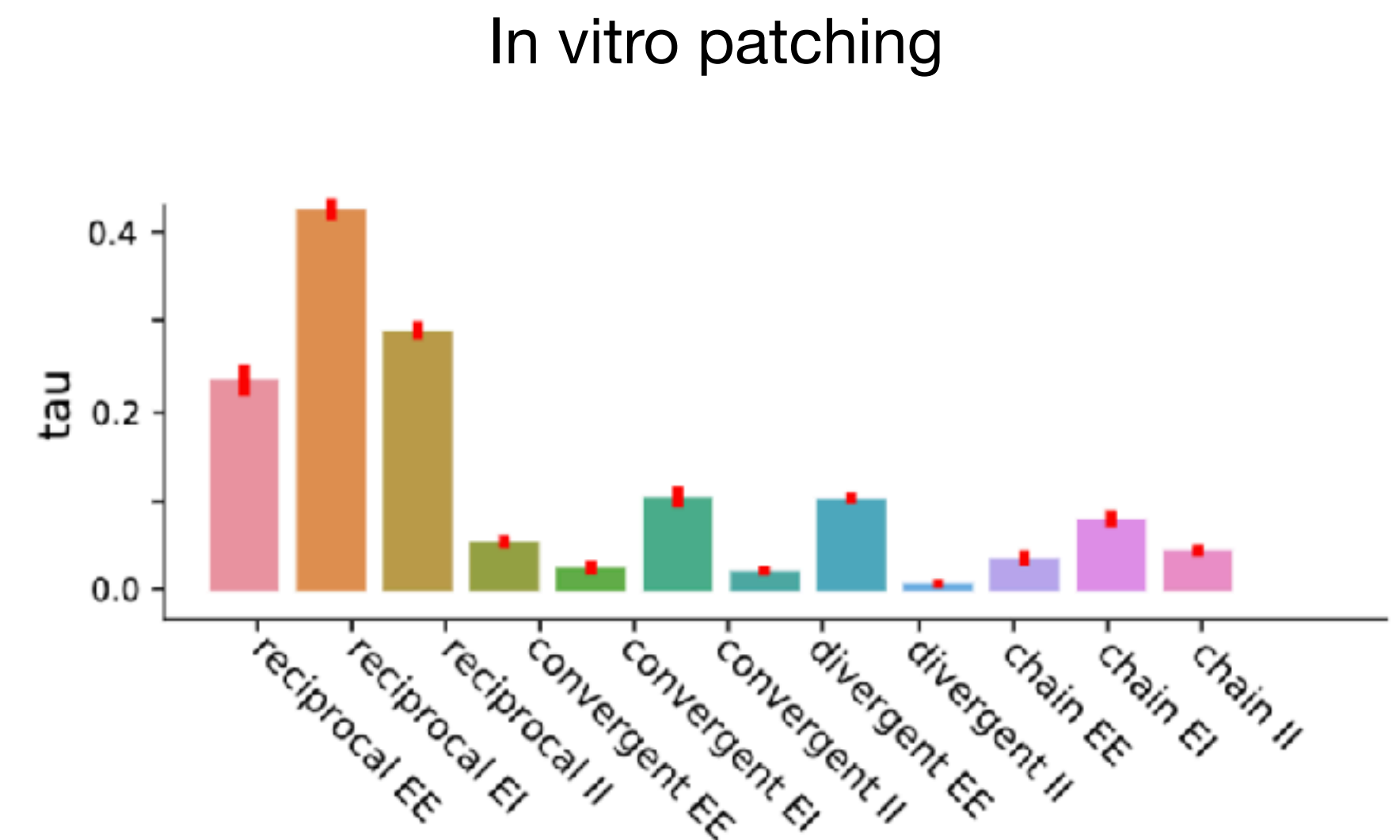
➔ **Mean-field breaks down!**

[Marti, Brunel & SO 2018]

Postsynaptic

	L2/3 Pyr spiny	L2/3 Pv	L2/3 Sst	L2/3 Vip
Presynaptic				
L2/3 Pyr spiny	13/229	22/53	20/67	11/68
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[Campagnola et al 2022]



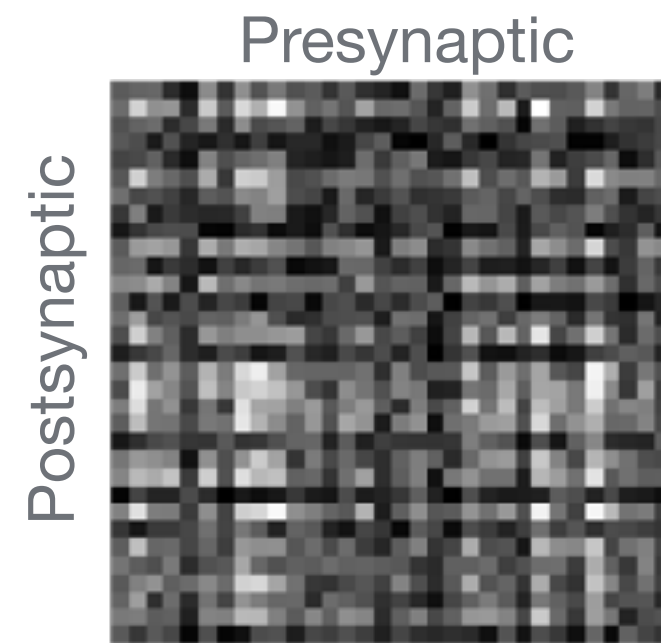
[Song et al 2005]

[Recanatesi et al 2020]

Global Connectivity

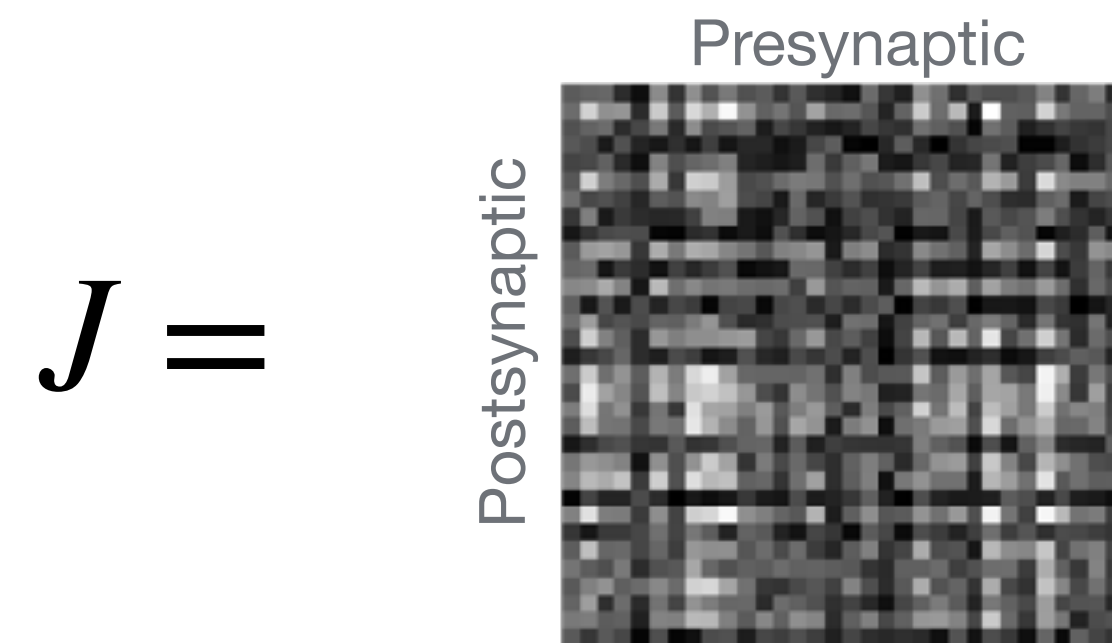
Connectivity matrix

$J =$

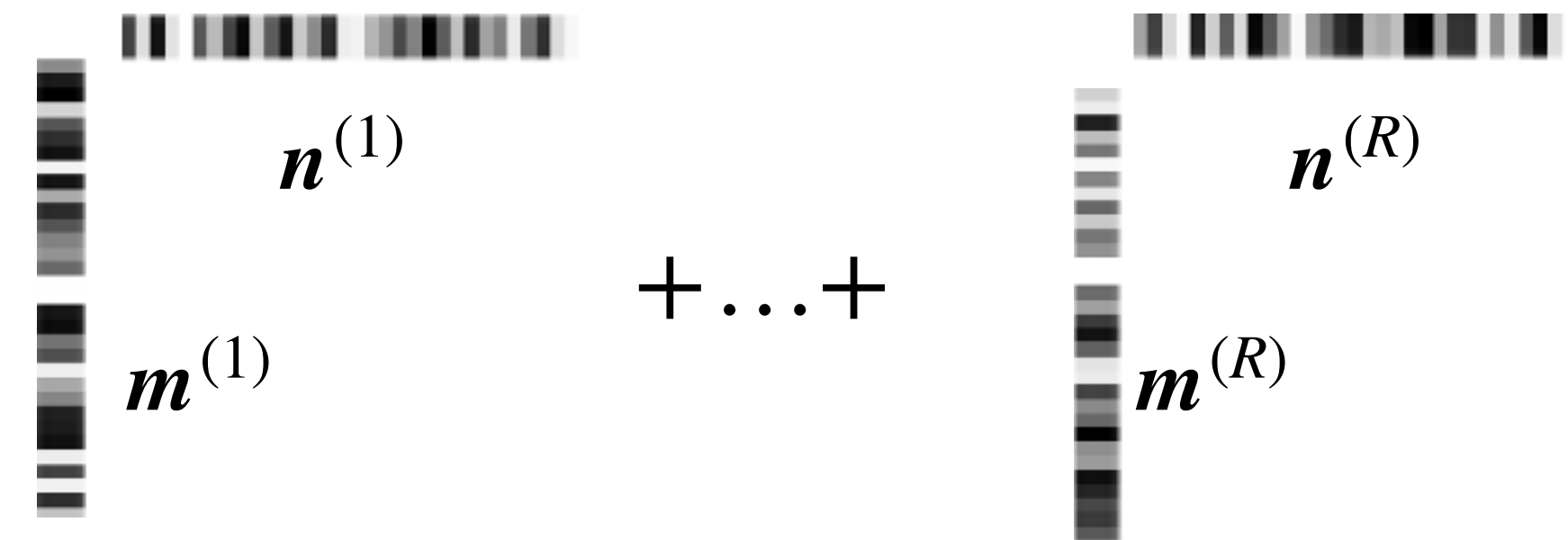


Global Connectivity

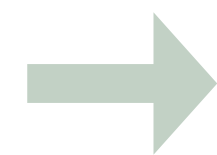
Connectivity matrix



=



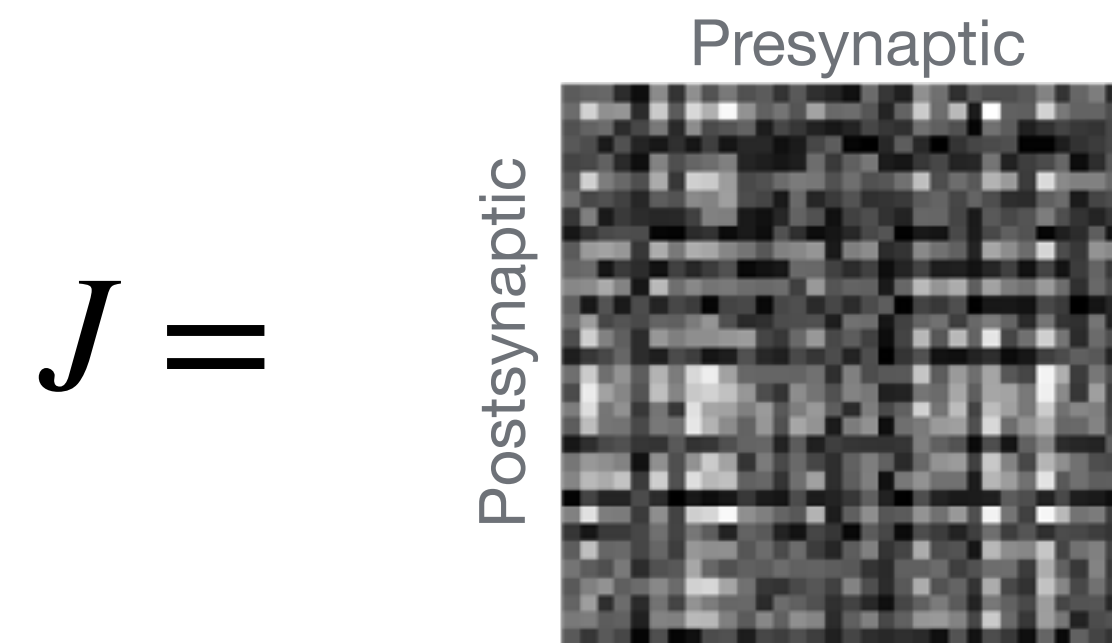
[Hopfield 1982, Ben Yishai etl al 1995, Seung 1996, Eliasmith 2005, Sussillo & Abbott 2009...]



Correlations

Global Connectivity

Connectivity matrix

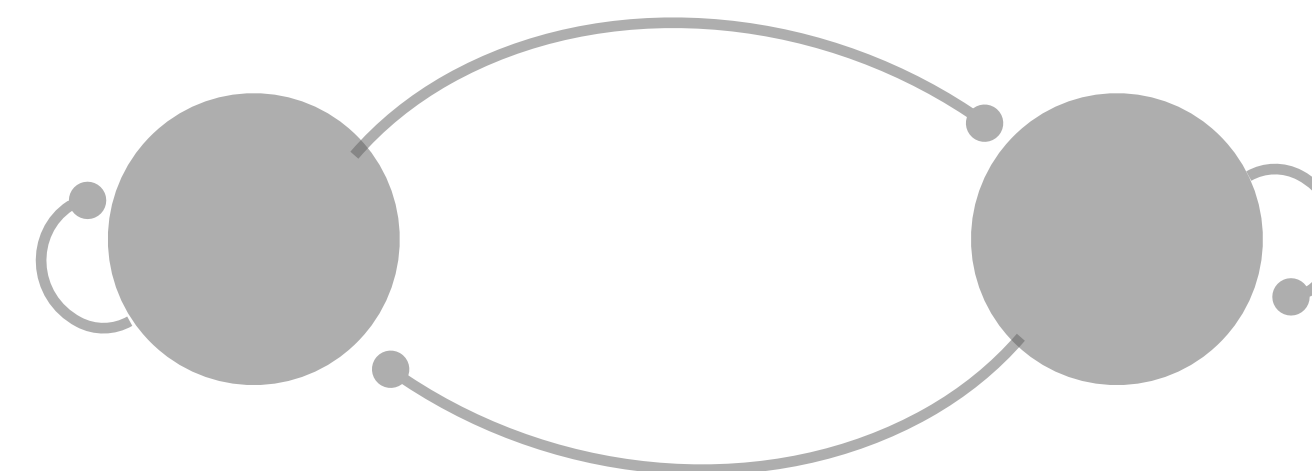


Low-rank

$$= \begin{matrix} \text{[row vector]} \\ m^{(1)} \end{matrix} \begin{matrix} n^{(1)} \\ \text{[column vector]} \end{matrix} + \dots + \begin{matrix} \text{[row vector]} \\ m^{(R)} \end{matrix} \begin{matrix} n^{(R)} \\ \text{[column vector]} \end{matrix}$$

[Hopfield 1982, Ben Yishai et al 1995, Seung 1996, Eliasmith 2005, Sussillo & Abbott 2009...]

Low-d dynamics



[Mastrogiuseppe & SO 2018, Beiran et al 2021, Dubreuil, Valente et al 2022 ...]

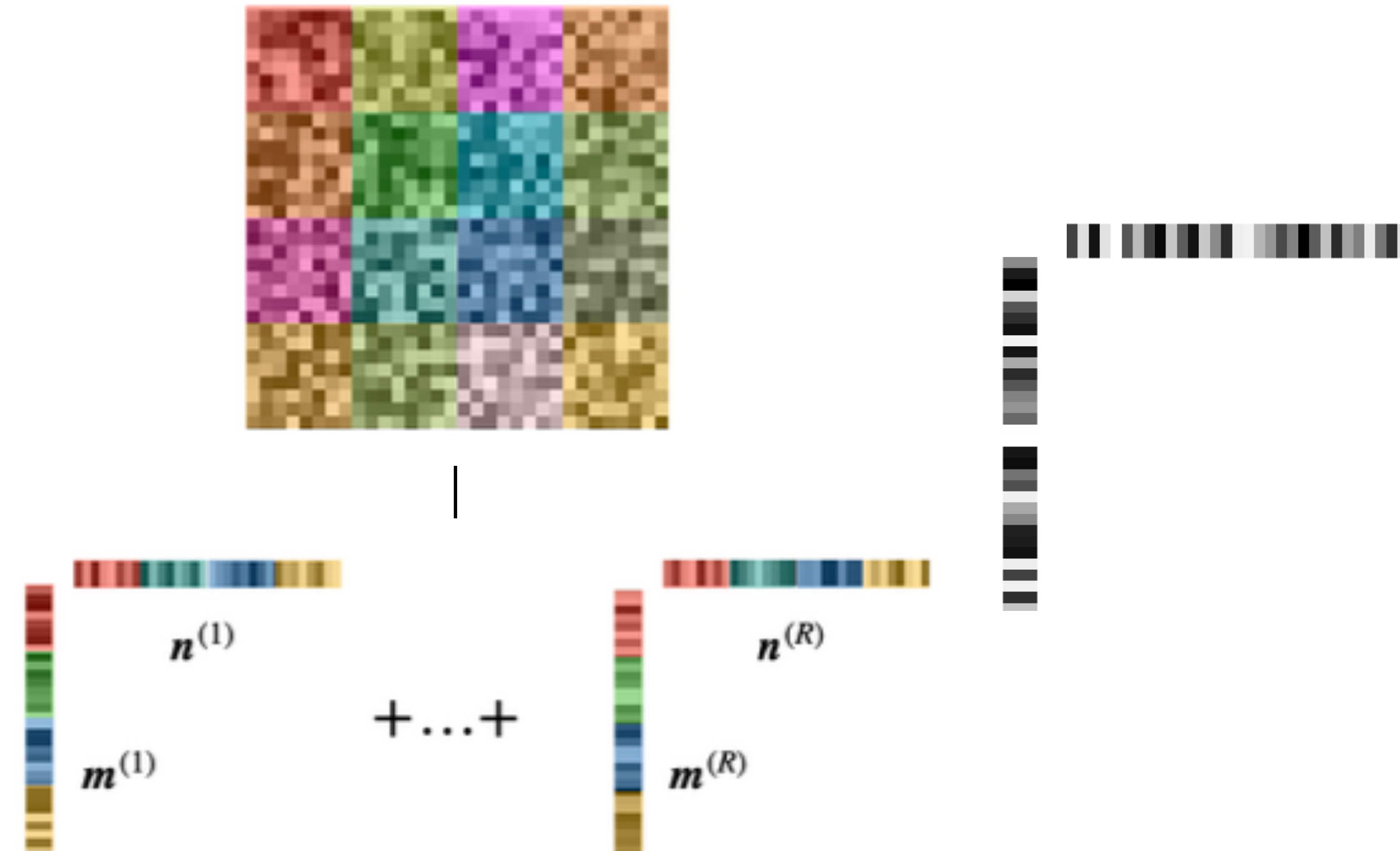


Effective model

Local Connectivity [EI networks]

	L2/3 Pyr spiny	L2/3 Pv	L2/3 Sst	L2/3 Vip
L2/3 Pyr spiny	13/229	22/53	20/67	11/68
L2/3 Pv	18/52	45/114	8/88	0/54
L2/3 Sst	13/56	15/84	8/154	25/84
L2/3 Vip	3/62	1/54	12/87	2/209

Global Connectivity [low-rank RNNs]



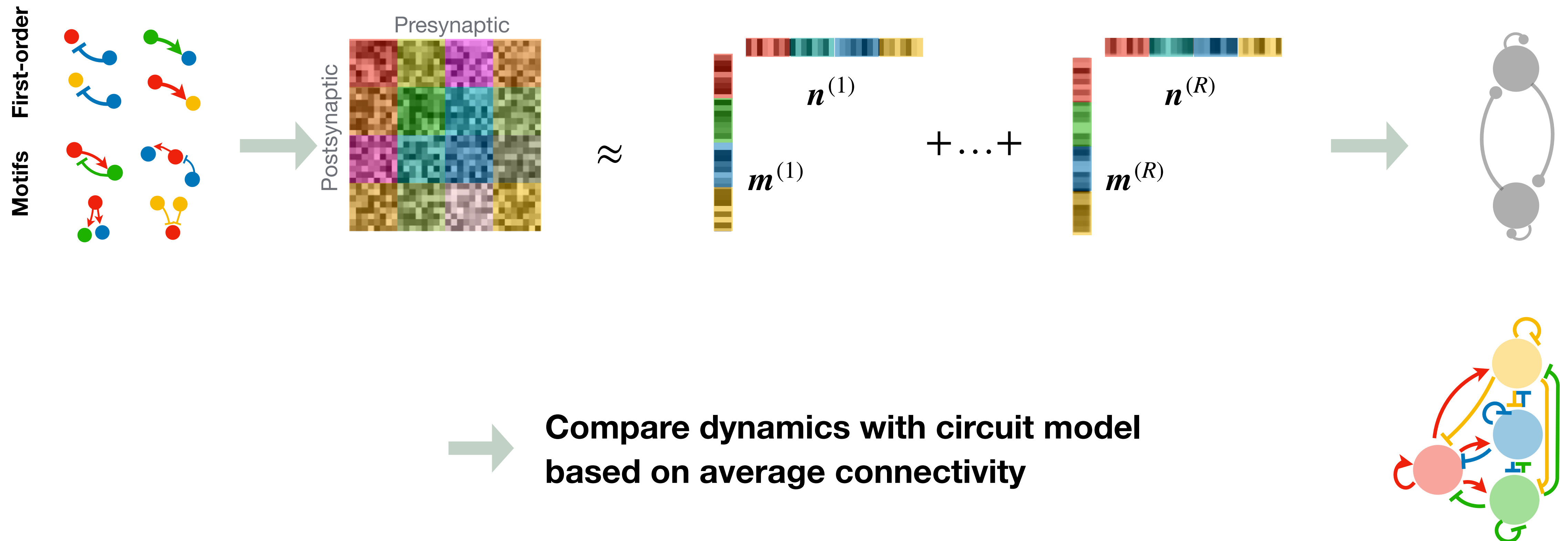
→ **Combine the two approaches**

Combining Local and Global Connectivity

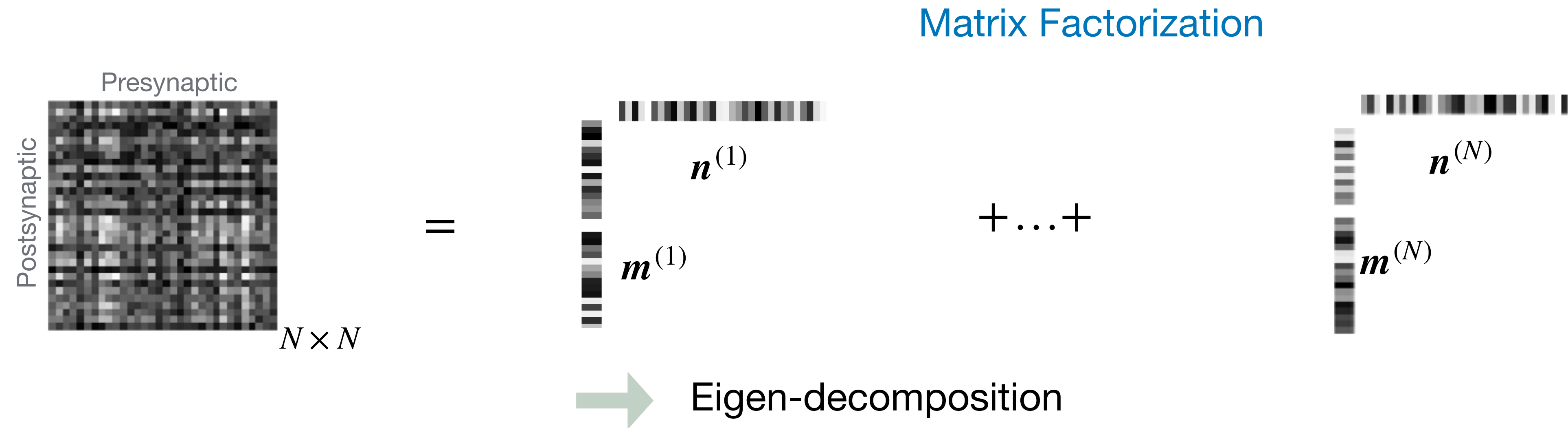
1. Define network from local connectivity based on multiple cell types

2. Approximate by low-rank network

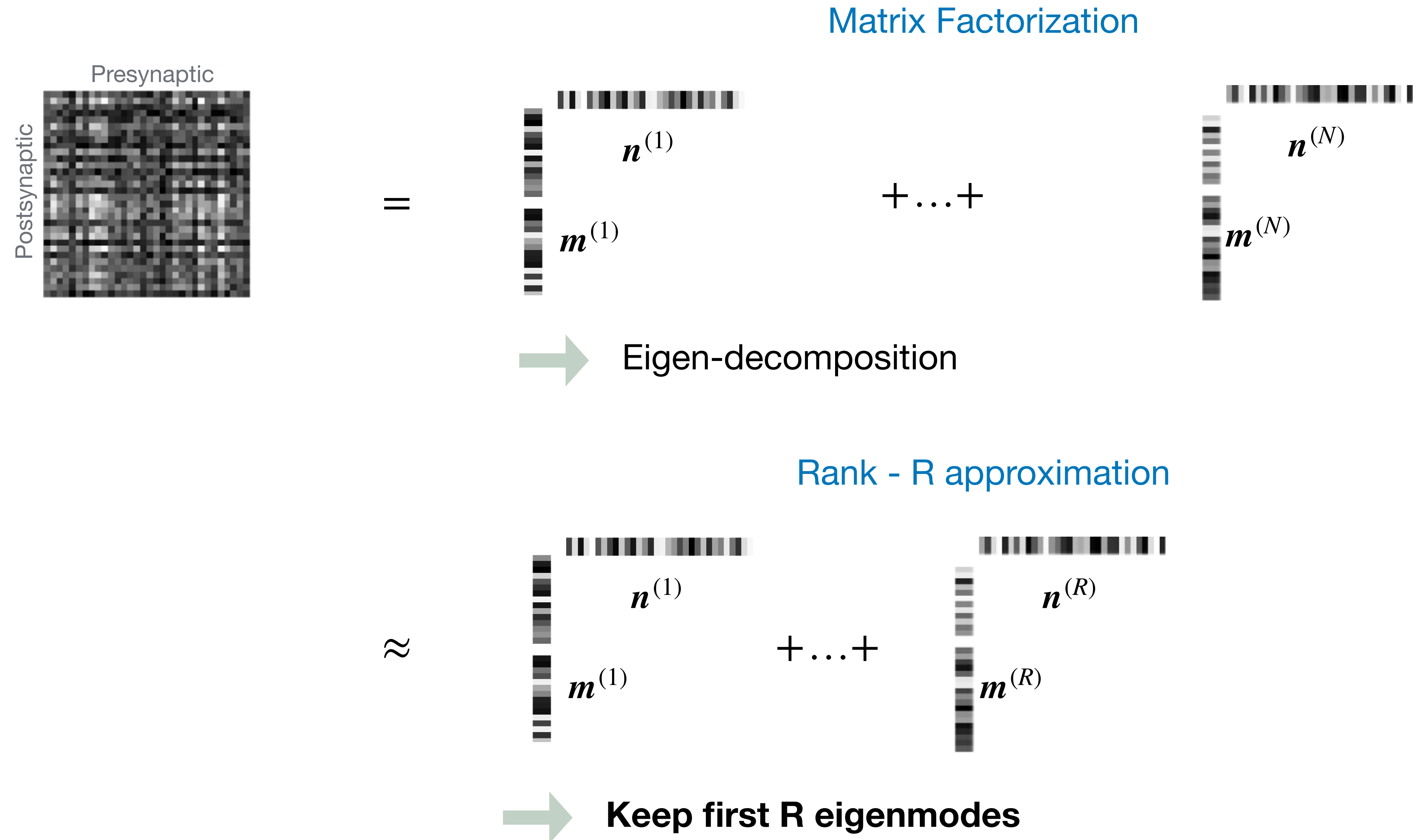
3. Reduce to low-D Model



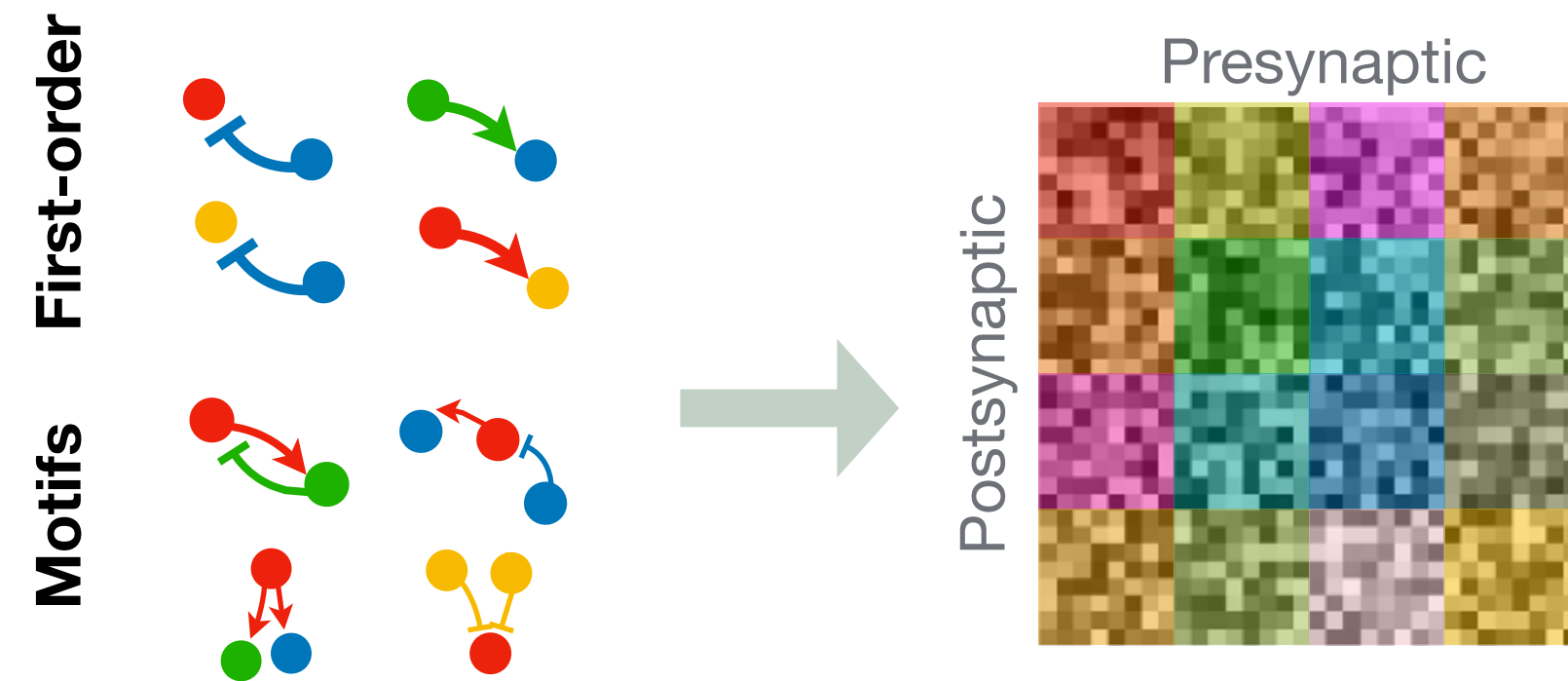
Approach: Low-rank Approximation



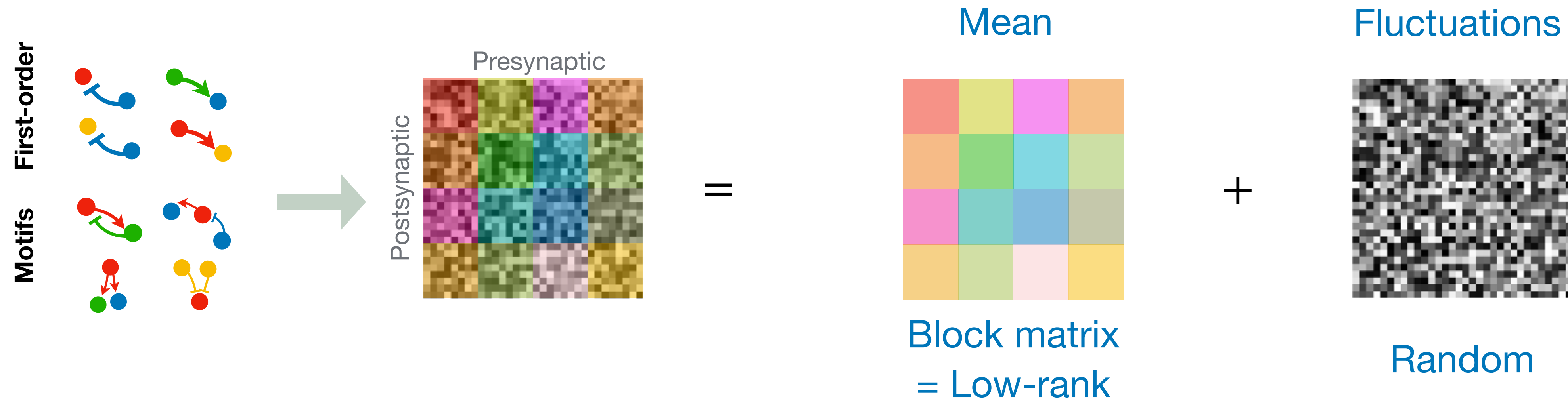
Approach: Low-rank Approximation



Approach: Low-rank Approximation



Approach: Low-rank Approximation



→ **Compute eigenvalues analytically**

Results

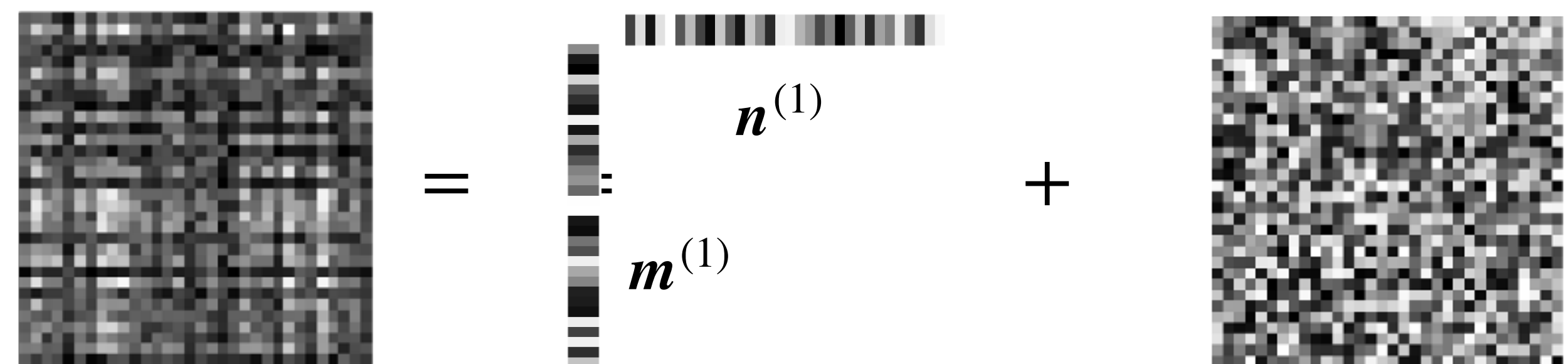
1. Methods: eigenvalues and eigenspectra
2. E-I networks: Eigenvalues and low-rank approximation
 - 2.1. Uncorrelated networks
 - 2.2. Reciprocal motifs
 - 2.3. Chain motifs
3. Dynamics: (paradoxical) responses to inputs

Results

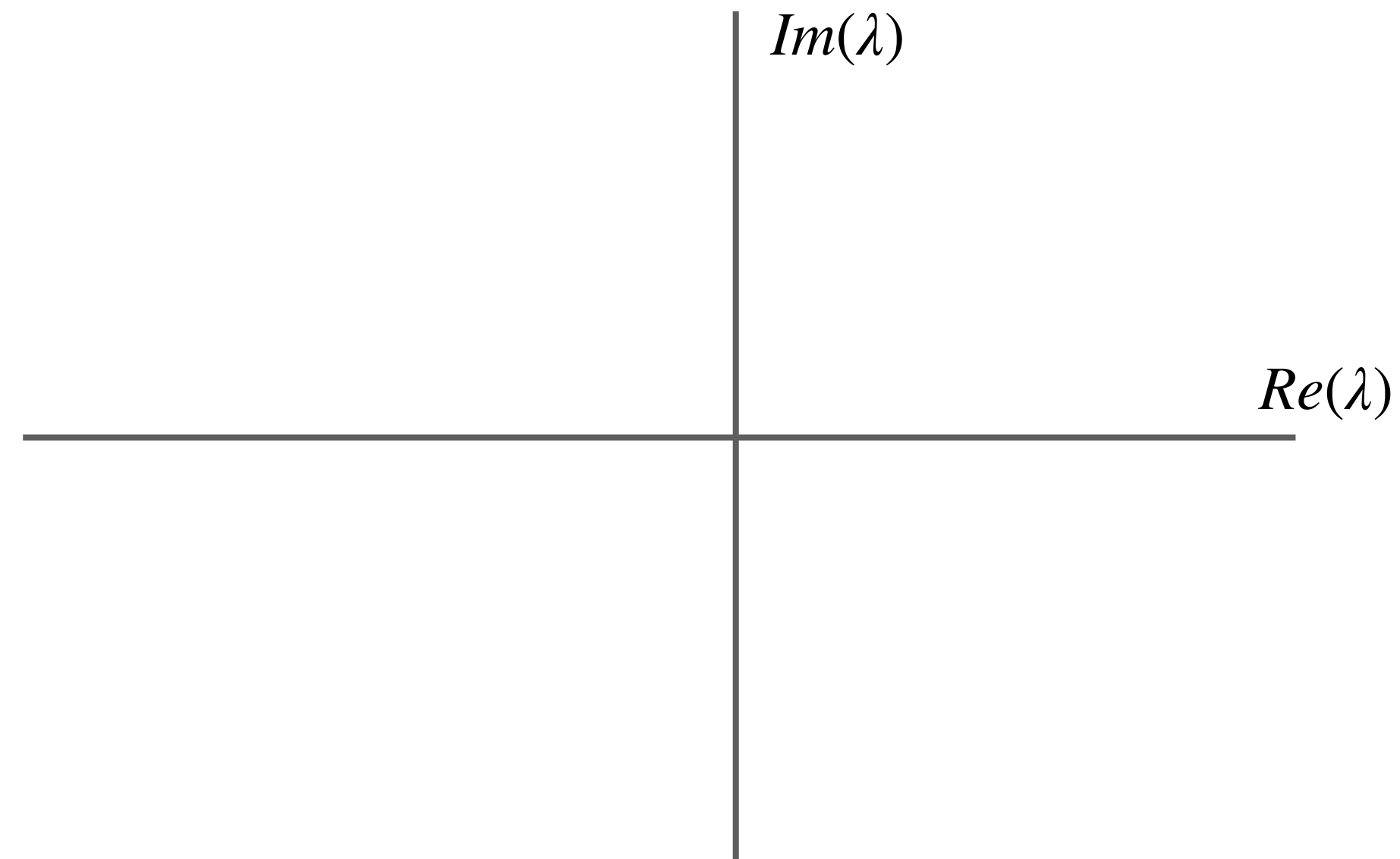
1. **Methods: eigenvalues and eigenspectra**
2. E-I networks: Eigenvalues and low-rank approximation
3. Dynamics: (paradoxical) responses to inputs

Methods: Eigenvalues and Eigenspectra

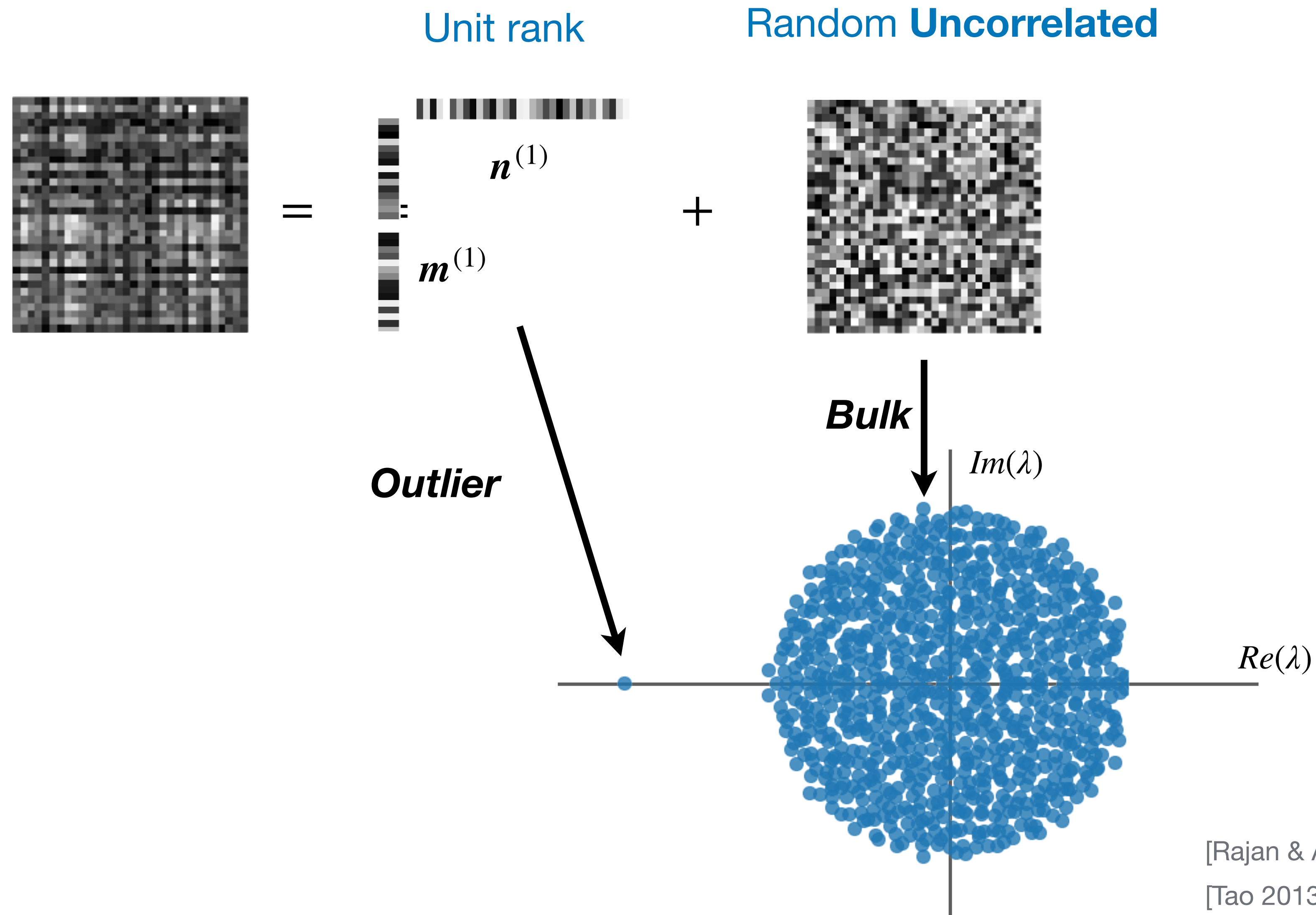
Unit rank Random **Uncorrelated**


$$\text{Matrix} = \begin{matrix} \text{Unit rank} \\ \text{Random **Uncorrelated**} \end{matrix} + \text{Matrix}$$

The diagram illustrates a matrix decomposition. On the left is a square matrix with a noisy, random-like pattern. This is followed by an equals sign. To the right of the equals sign is a unit rank matrix, represented by a vertical vector of horizontal bars labeled $m^{(1)}$ and a horizontal vector of vertical bars labeled $n^{(1)}$. This is followed by a plus sign and another square matrix, which is labeled 'Random **Uncorrelated**' and also has a noisy, random-like pattern.



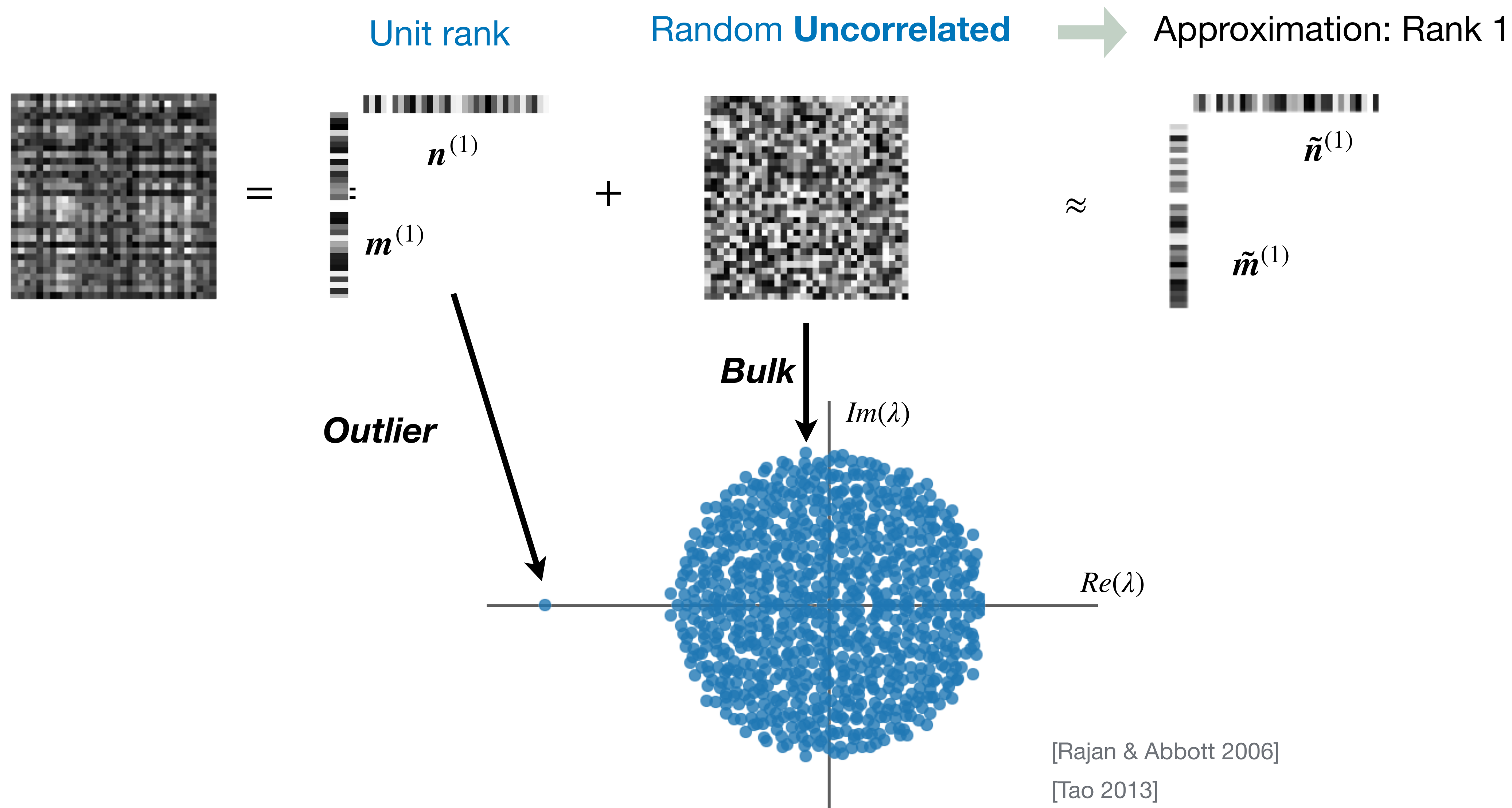
Methods: Eigenvalues and Eigenspectra



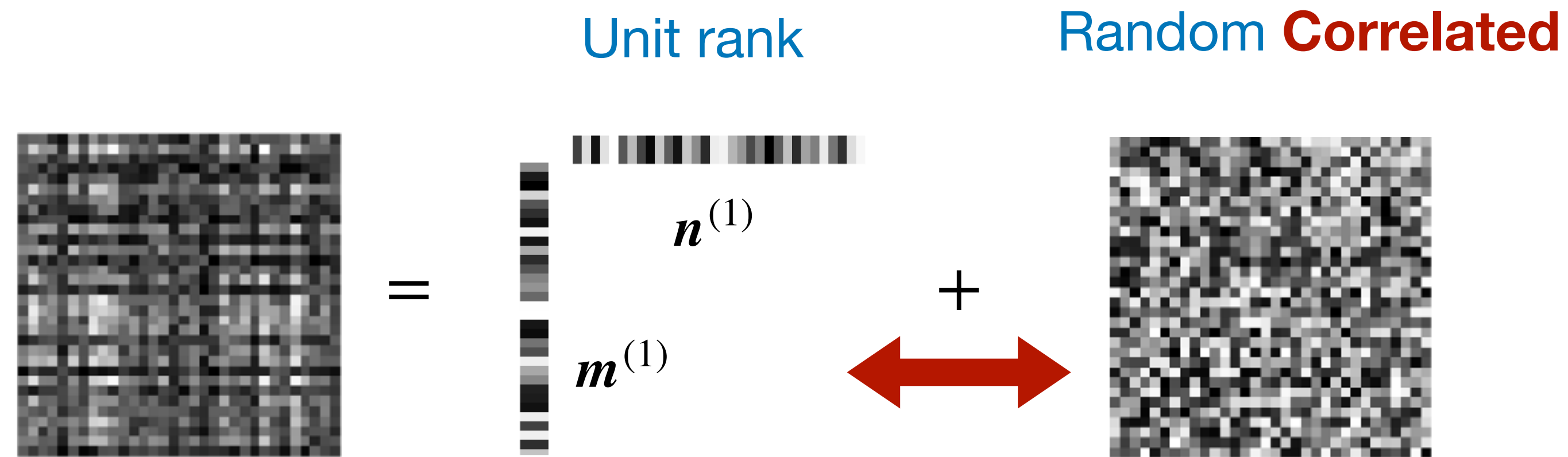
[Rajan & Abbott 2006]

[Tao 2013]

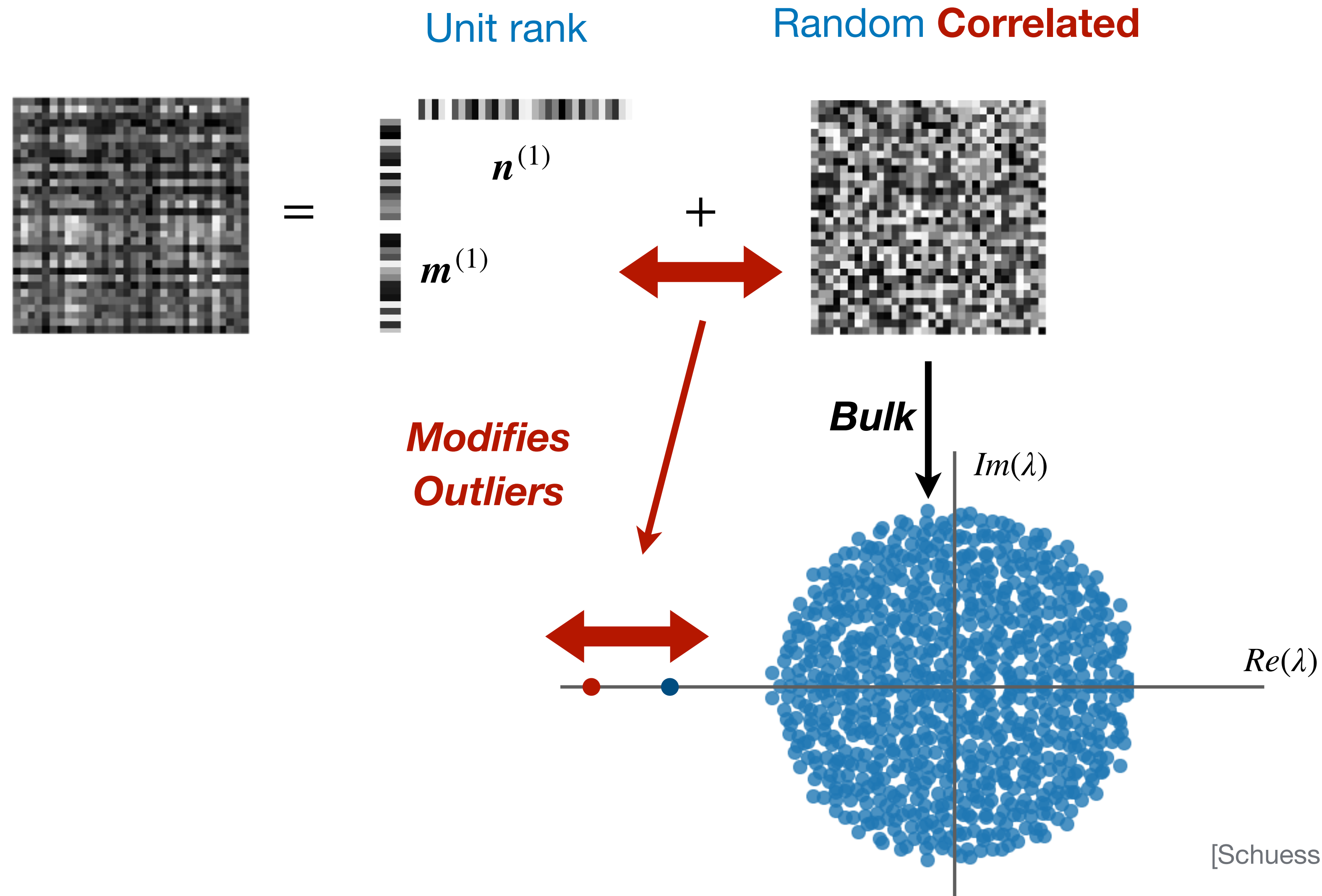
Methods: Eigenvalues and Eigenspectra



Methods: Eigenvalues and Eigenspectra

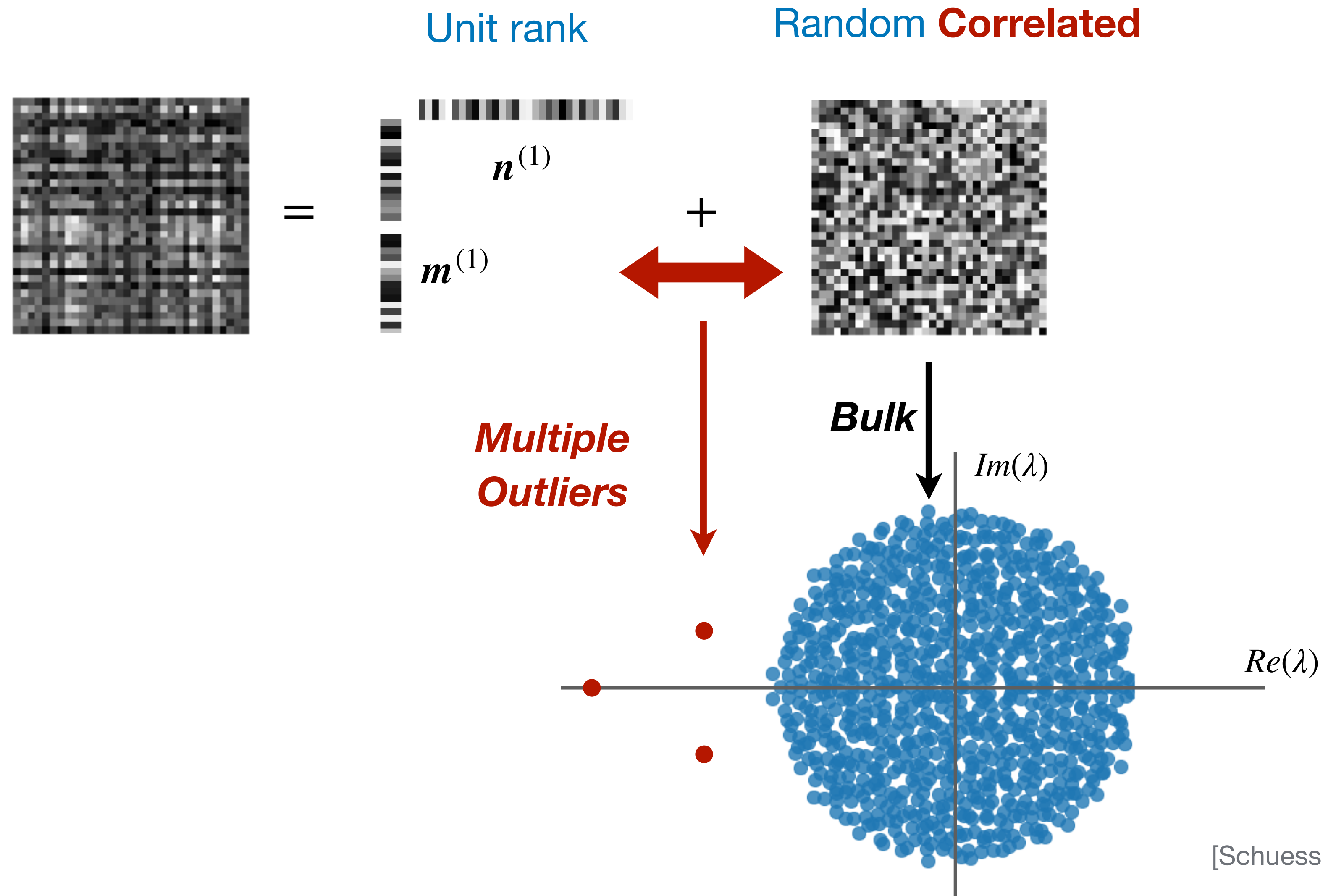


Methods: Eigenvalues and Eigenspectra

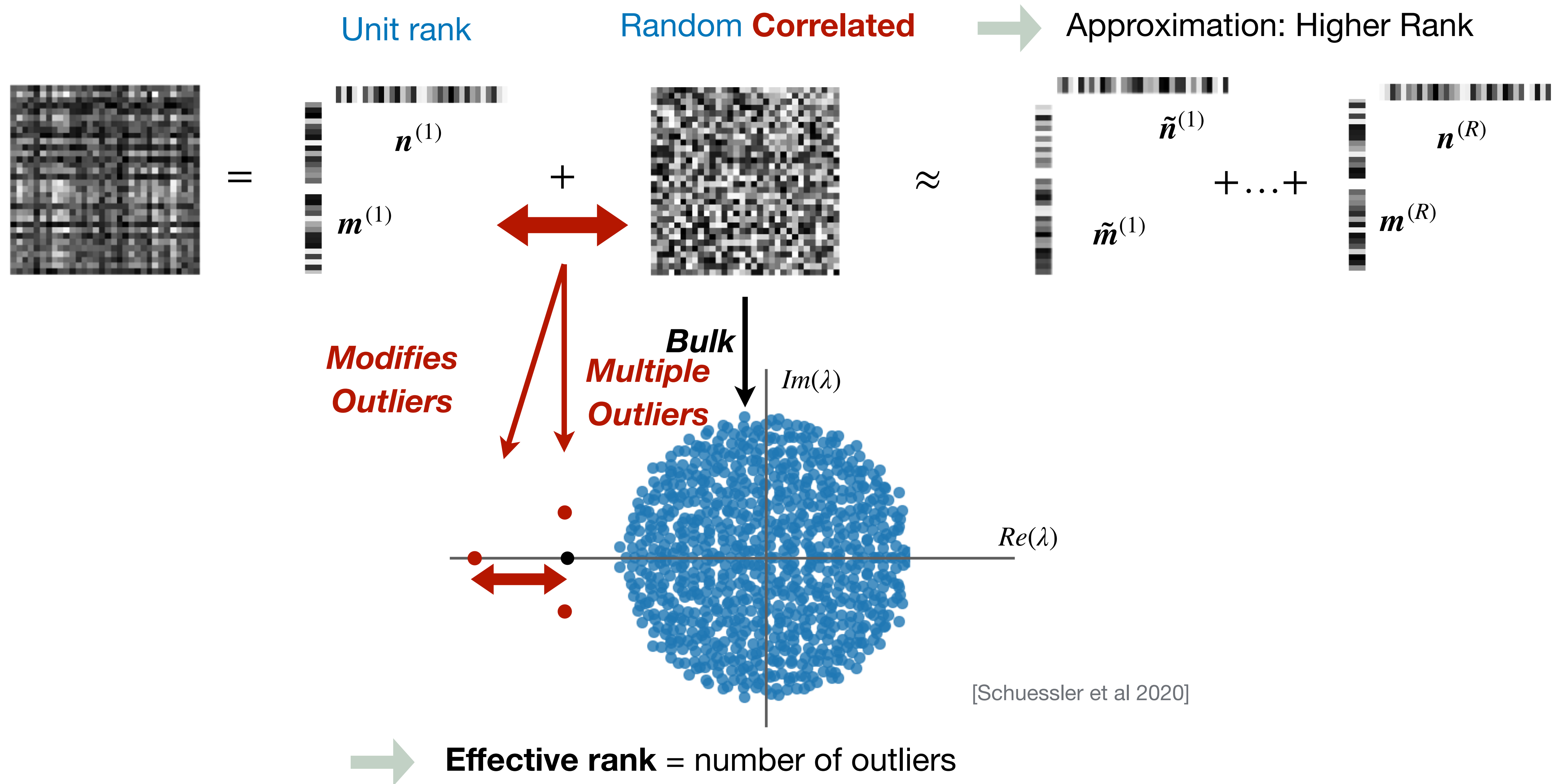


[Schuessler et al 2020]

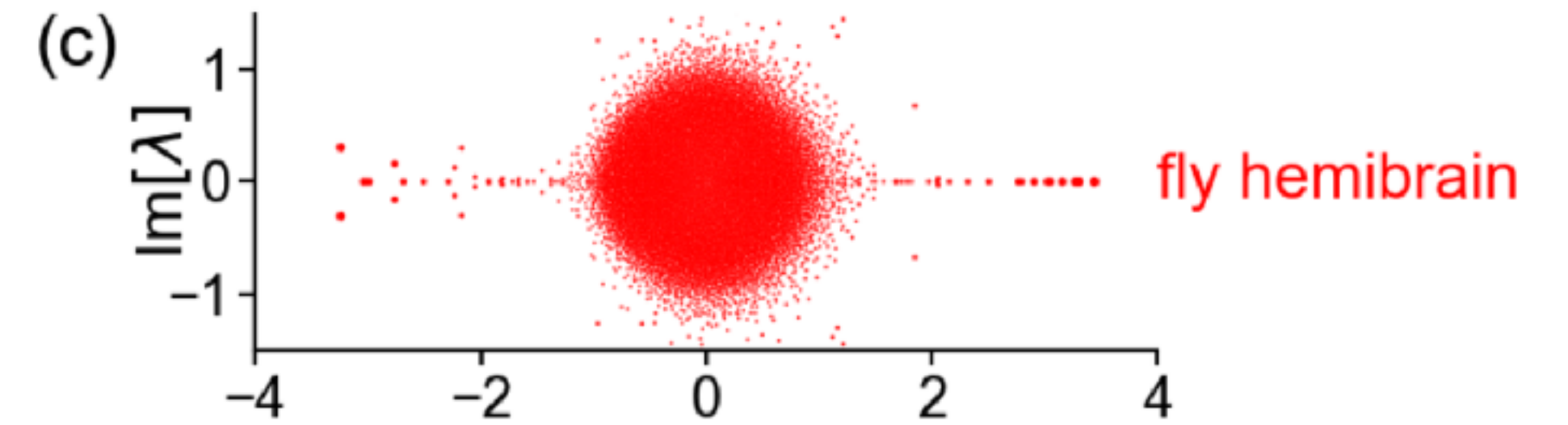
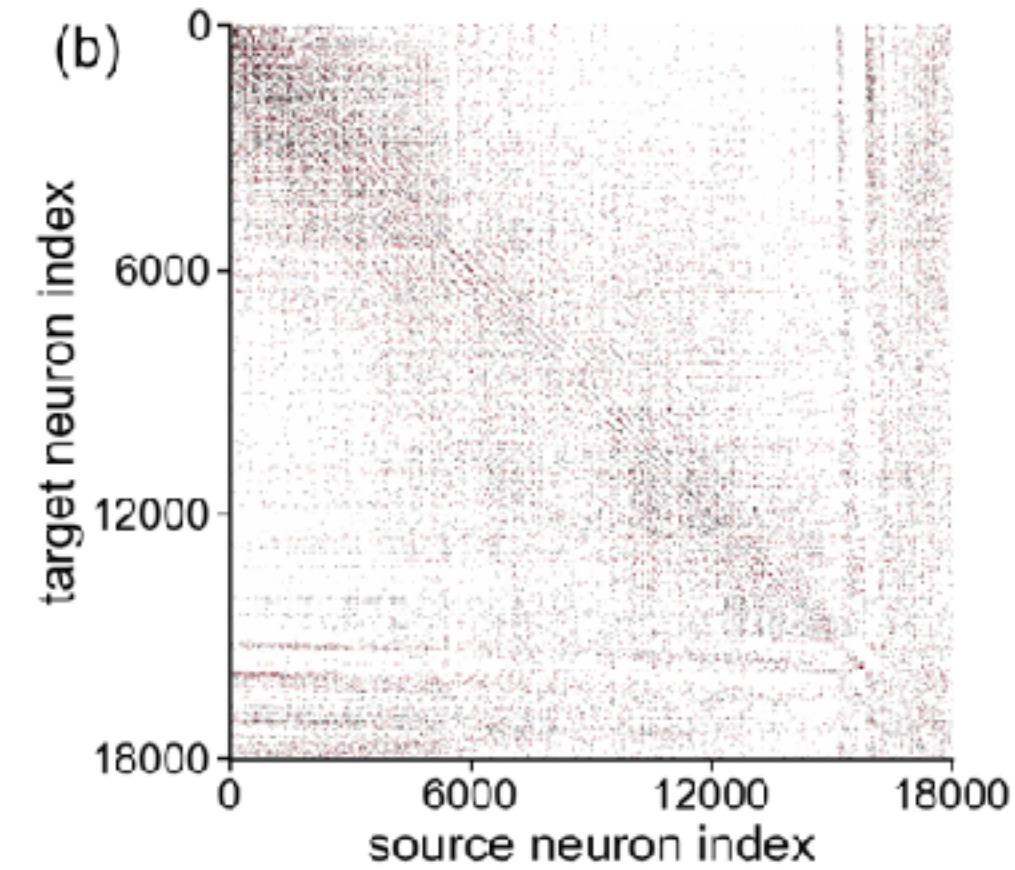
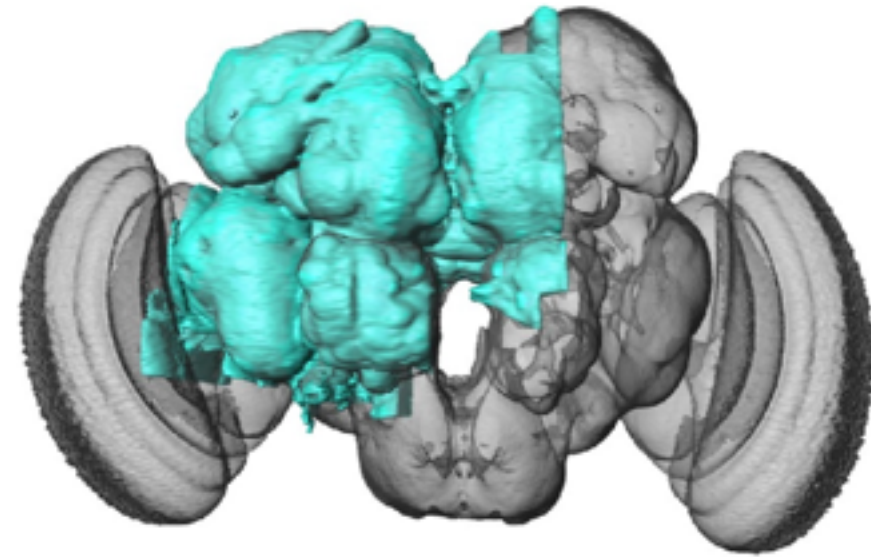
Methods: Eigenvalues and Eigenspectra



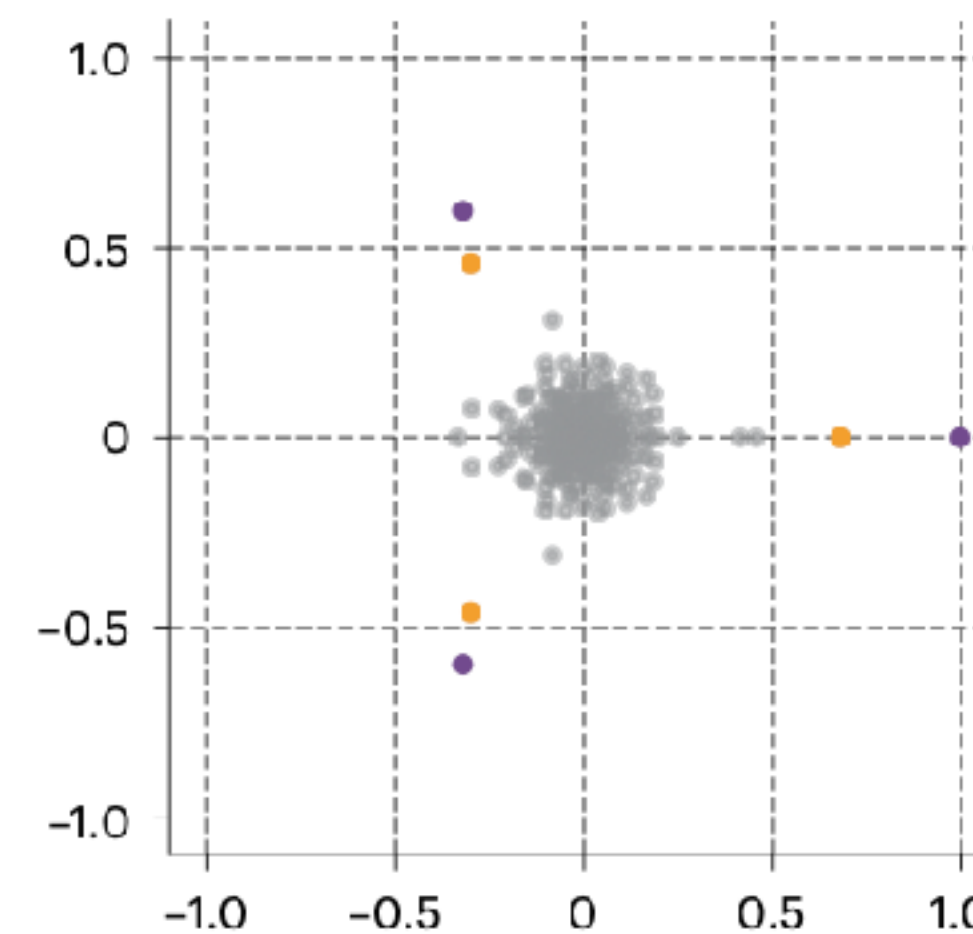
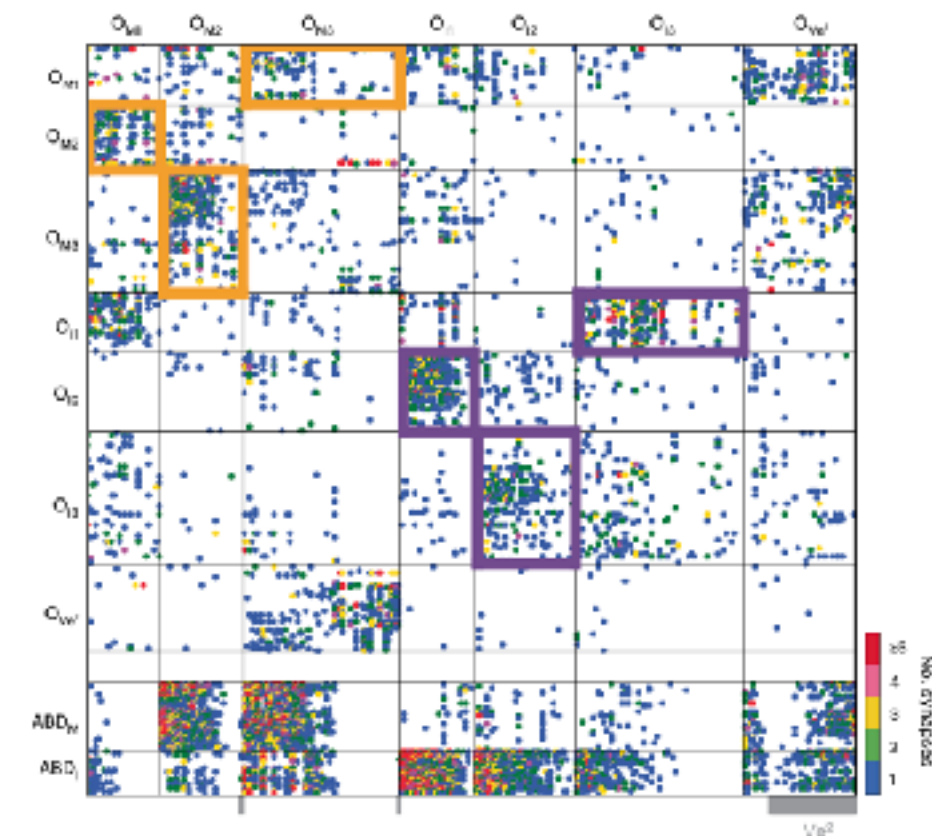
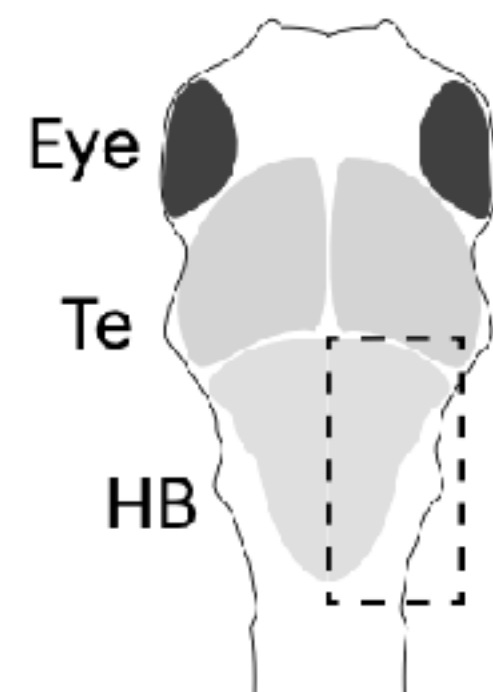
Methods: Eigenvalues and Eigenspectra



Global Connectivity: Experiments



[Clark et al 2024]



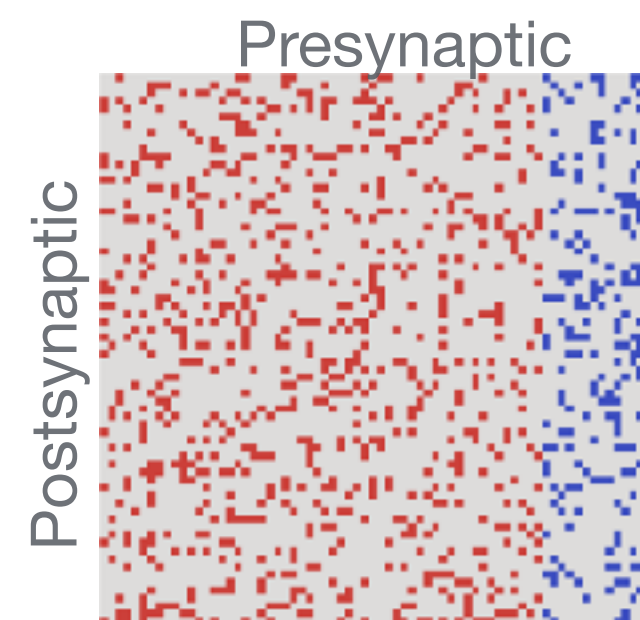
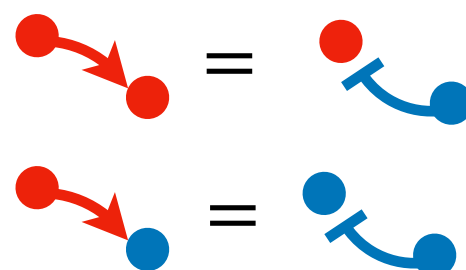
[Vishwanathan et al 2024]

Results

1. Methods: eigenvalues and eigenspectra
- 2. E-I networks: Eigenvalues and low-rank approximation**
 - 2.1. Uncorrelated networks
 - 2.2. Reciprocal motifs
 - 2.3. Chain motifs
3. Dynamics: (paradoxical) responses to inputs

Simplified E-I network, no motifs

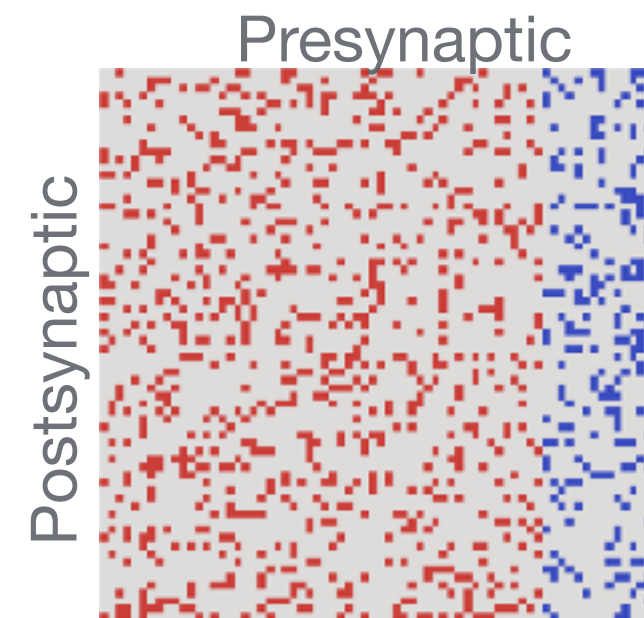
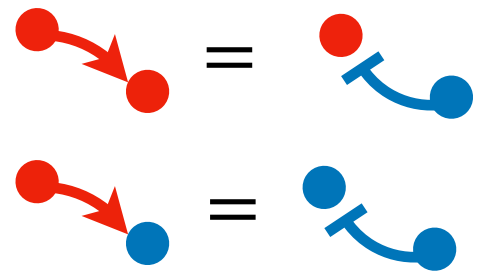
First-order



[Brunel 2000]

Simplified E-I network, no motifs

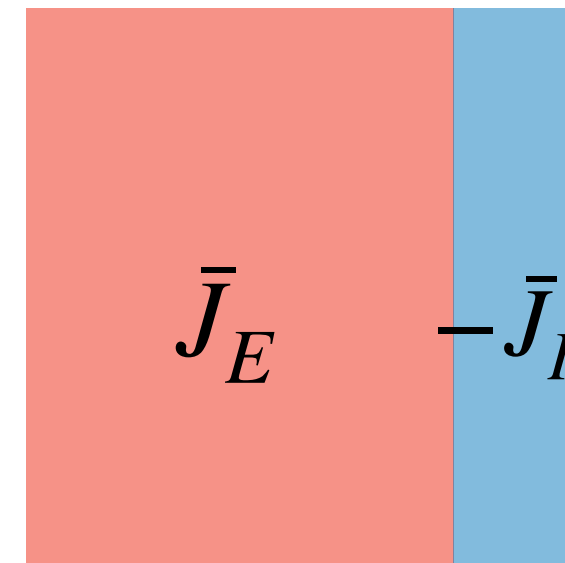
First-order



[Brunel 2000]

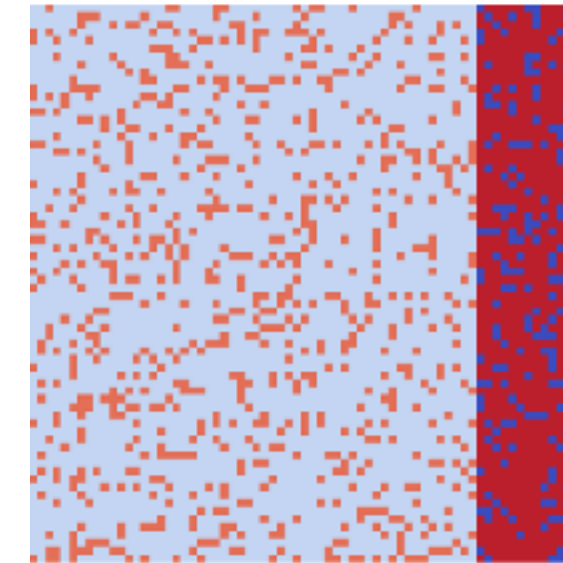
=

Mean



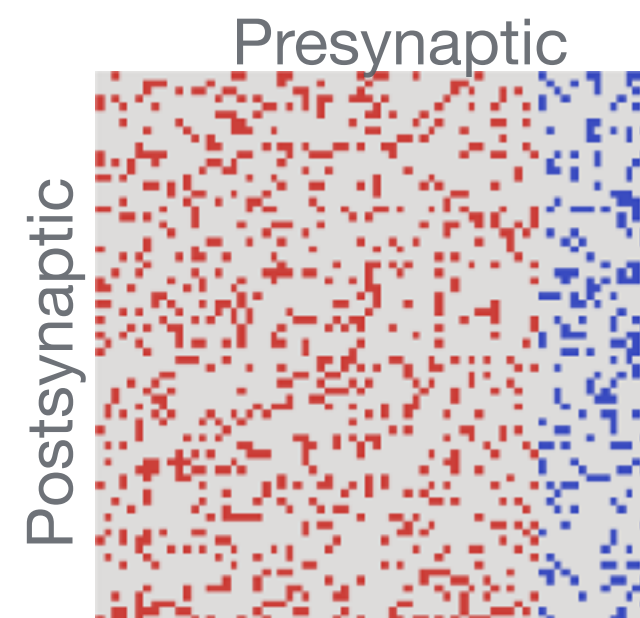
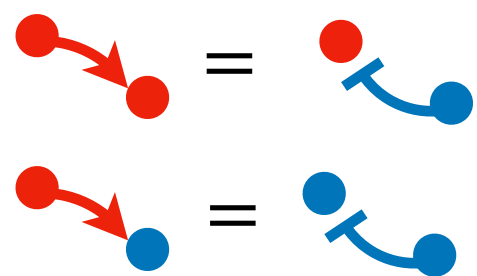
+

Fluctuations



Simplified E-I network, no motifs

First-order



[Brunel 2000]

=

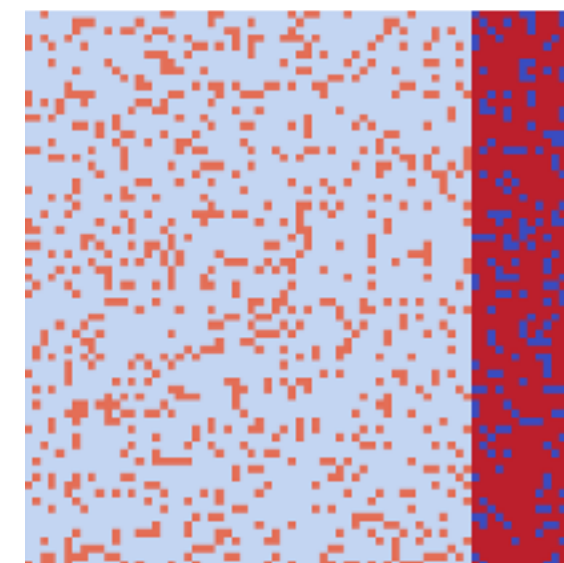
Unit Rank



$\bar{J}_E$ $-\bar{J}_I$

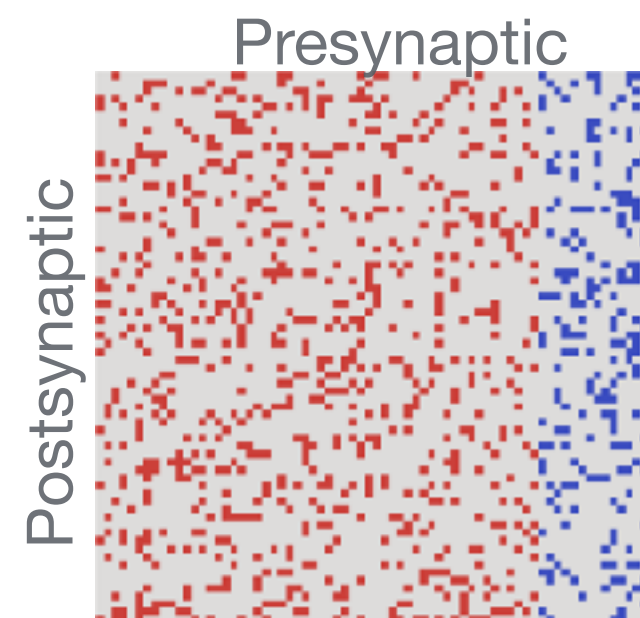
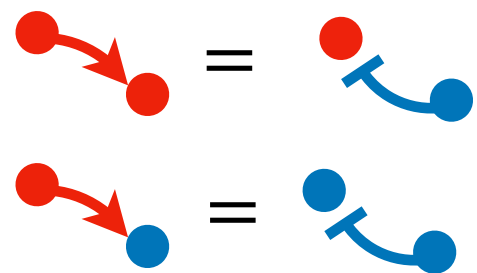
+

Random **Uncorrelated**



Simplified E-I network, no motifs

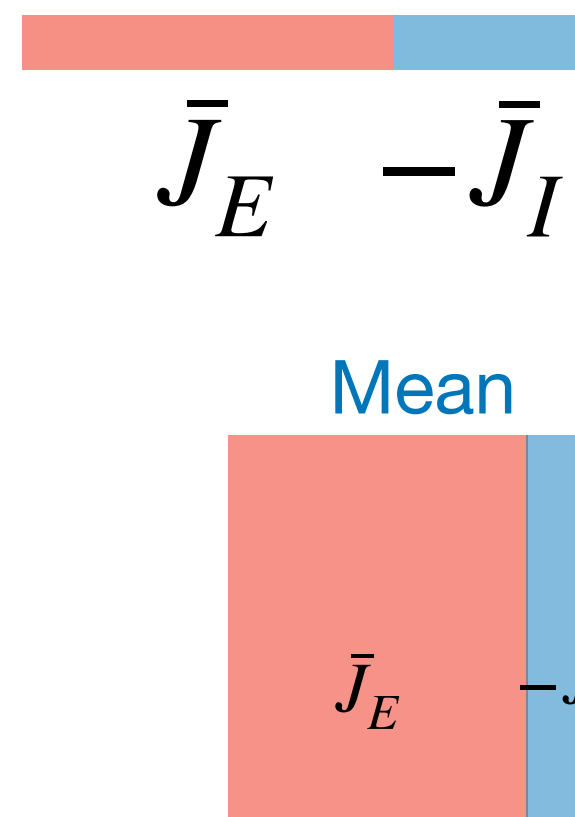
First-order



[Brunel 2000]

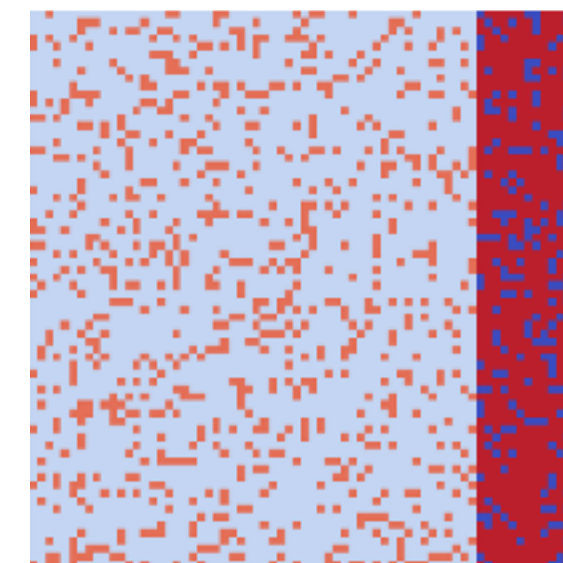
=

Unit Rank



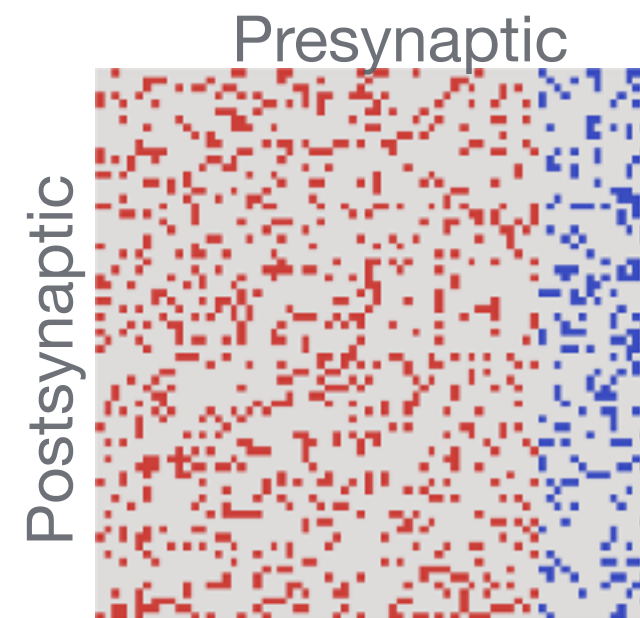
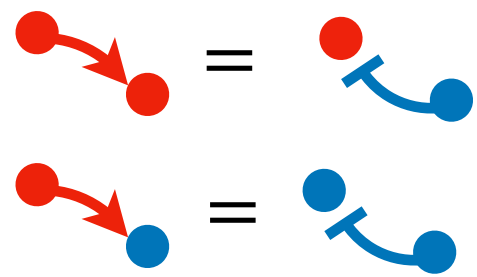
+

Random **Uncorrelated**



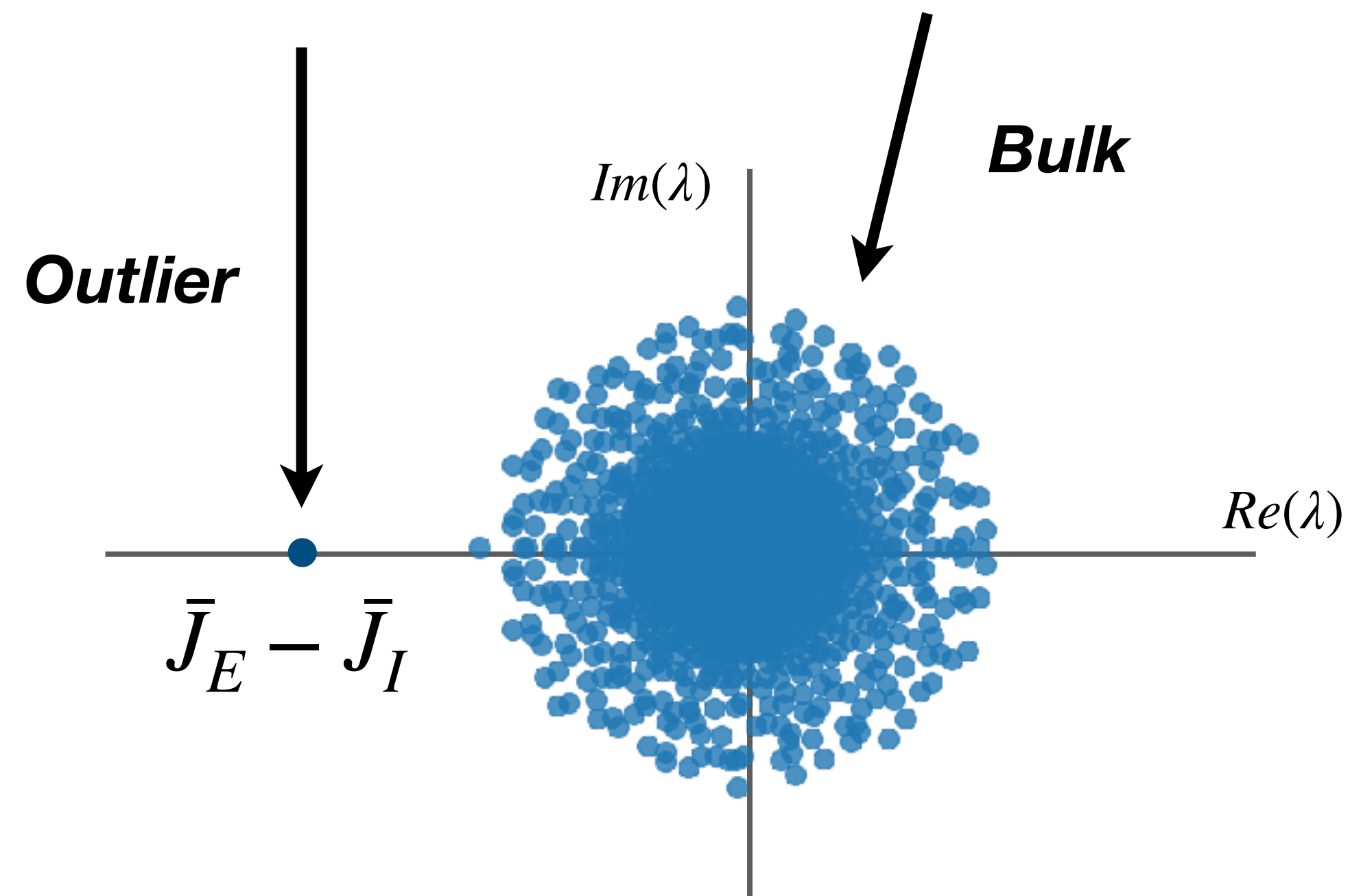
Simplified E-I network, no motifs

First-order



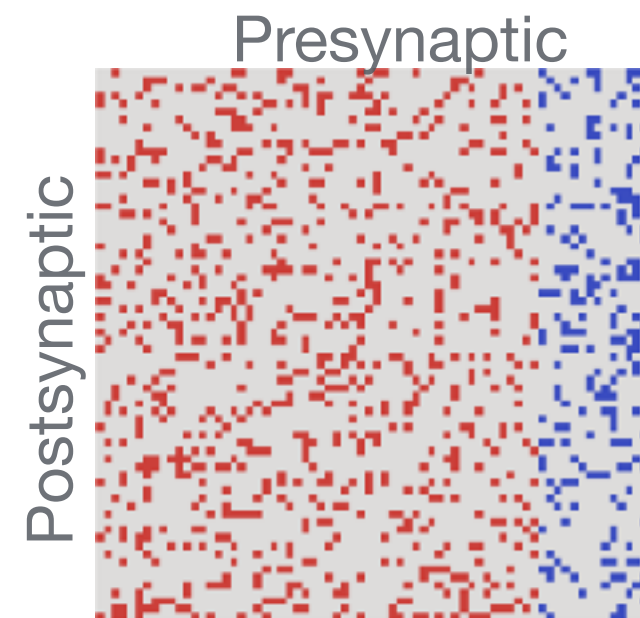
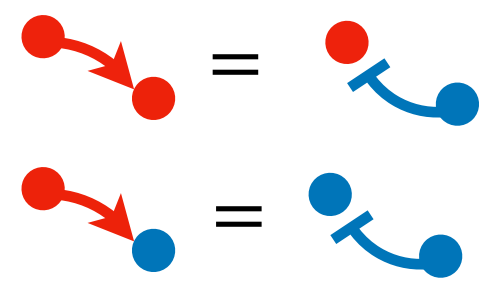
[Brunel 2000]

$$= \begin{matrix} \text{Unit Rank} \\ \bar{J}_E - \bar{J}_I \end{matrix} + \begin{matrix} \text{Random Uncorrelated} \end{matrix}$$



Simplified E-I network, no motifs

First-order



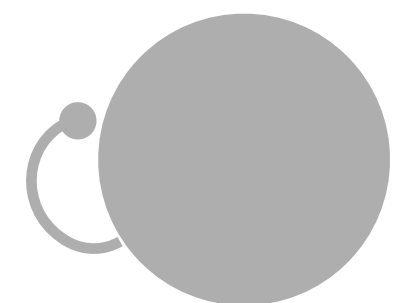
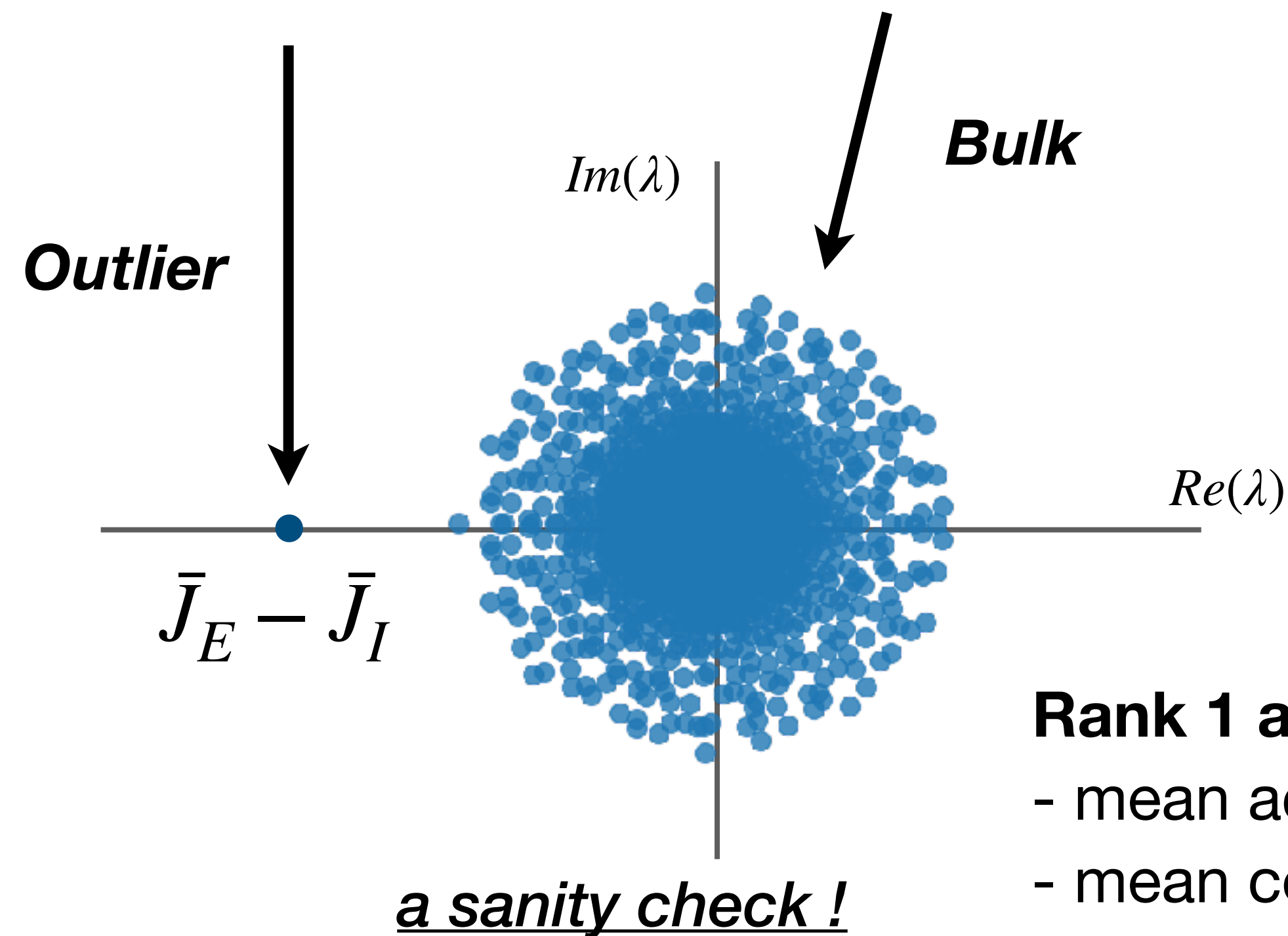
[Brunel 2000]

Unit Rank

Random Uncorrelated

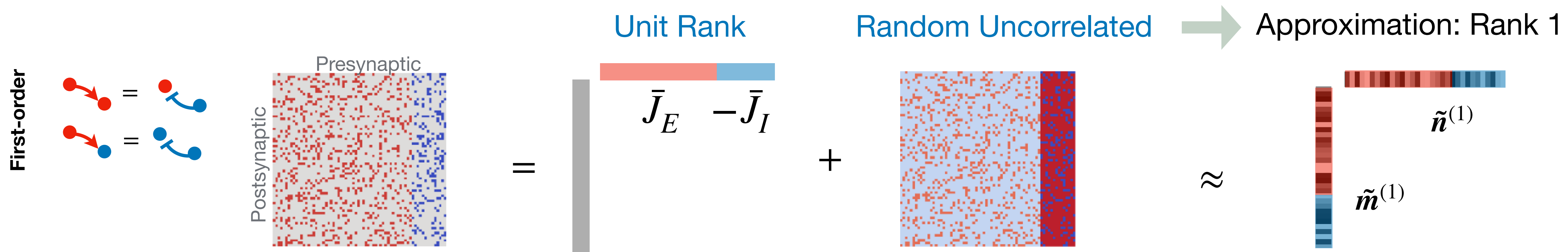
Approximation: Rank 1

$$= \begin{bmatrix} \bar{J}_E & -\bar{J}_I \end{bmatrix} + \begin{bmatrix} \text{Random Uncorrelated} \end{bmatrix} \approx \begin{bmatrix} \tilde{m}^{(1)} & \tilde{n}^{(1)} \end{bmatrix}$$



Rank 1 approximation: 1d model
 - mean activity
 - mean connectivity (+ variance)

Simplified E-I network, no motifs



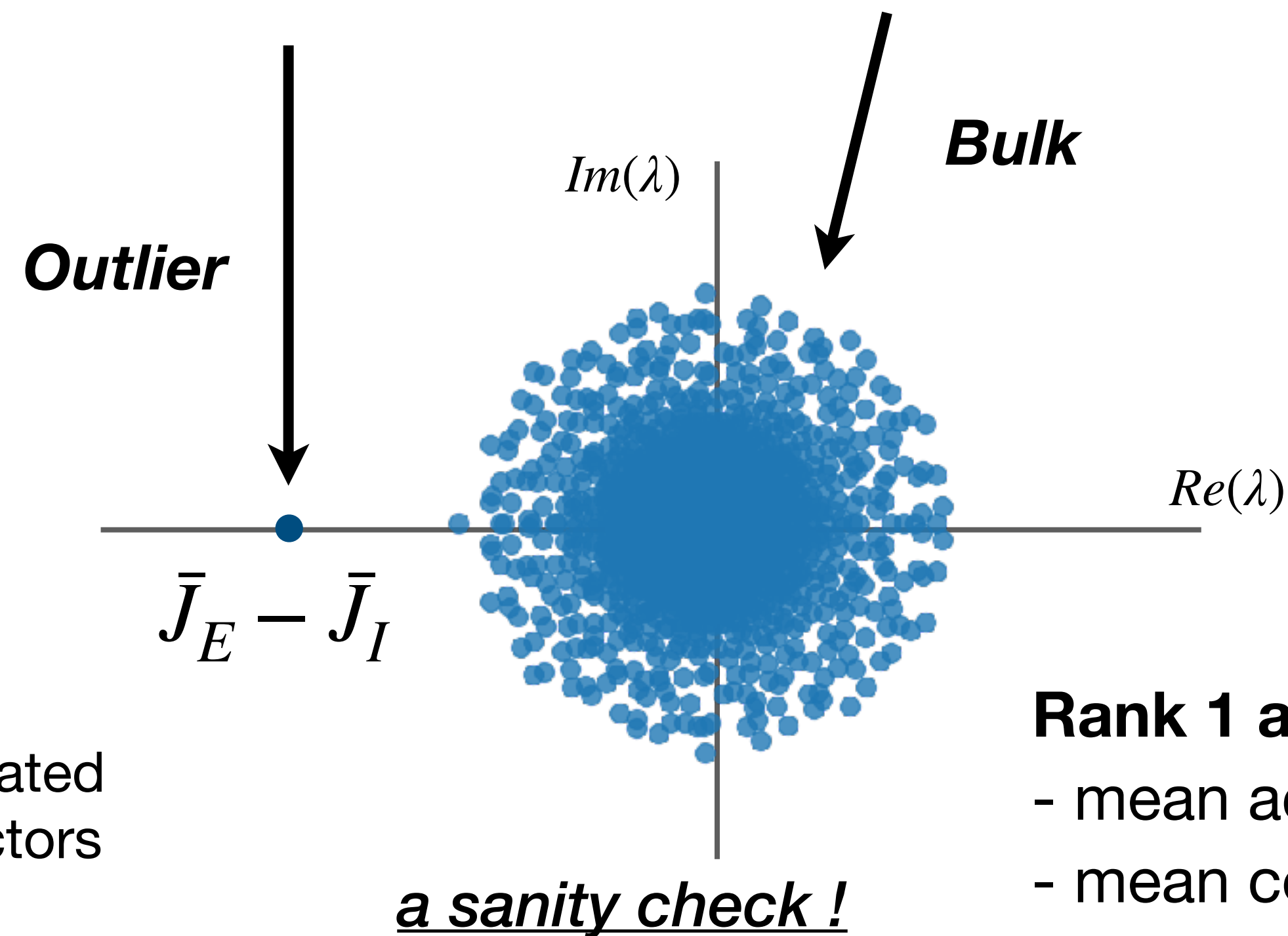
[Brunel 2000]

Works on the white board!

$$[\theta_k] = \frac{1}{N} \bar{\mathbf{n}}^T [\mathbf{Z}^k] \bar{\mathbf{m}} = 0$$

$$f(\lambda) = \lambda^3 - (\lambda_0 \lambda^2 + \theta_1 \lambda + \theta_2) = 0$$

i. i. d. local statistics $\rightarrow$ $[\lambda] = \lambda_0$ unchanged $\rightarrow$ Global $\rightarrow$ Uncorrelated eigenvectors

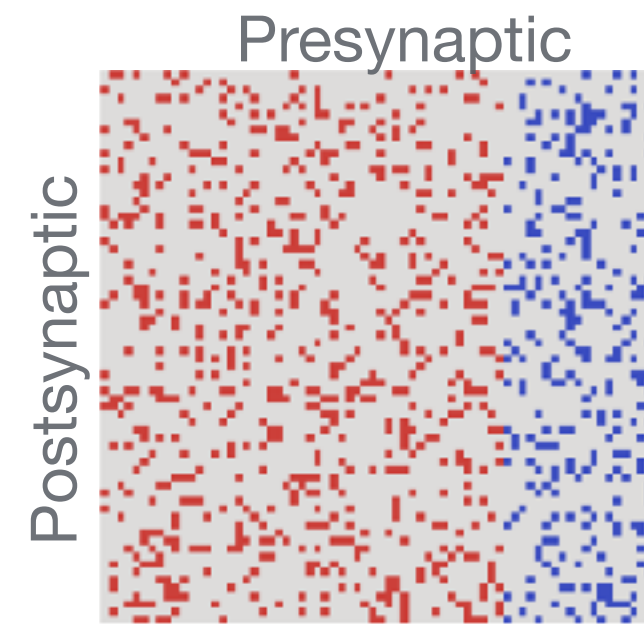
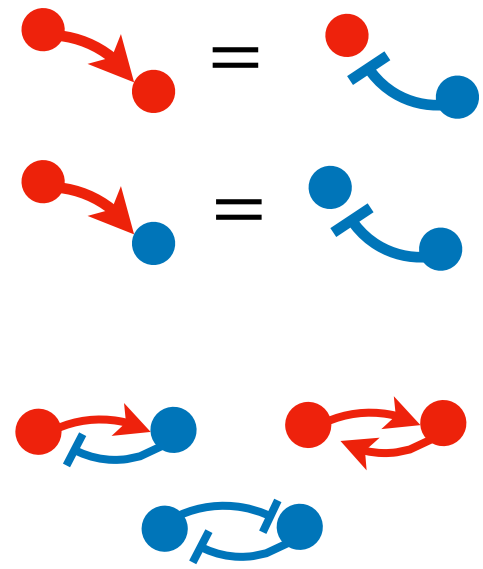


Rank 1 approximation: 1d model

- mean activity
- mean connectivity (+ variance)

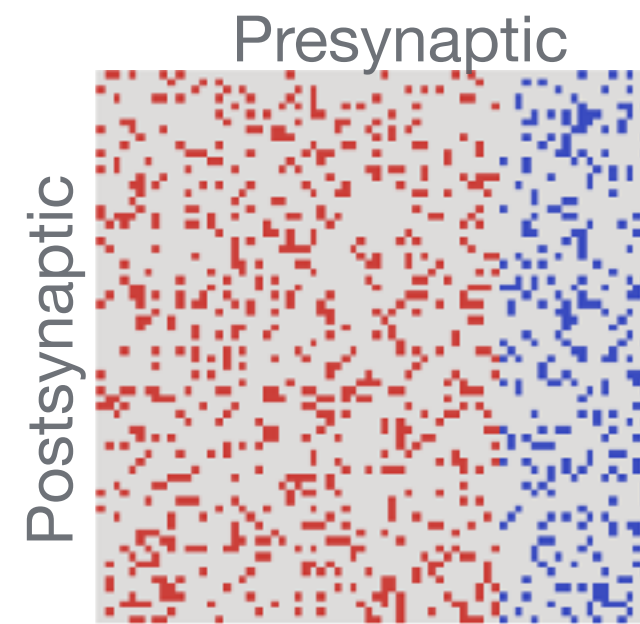
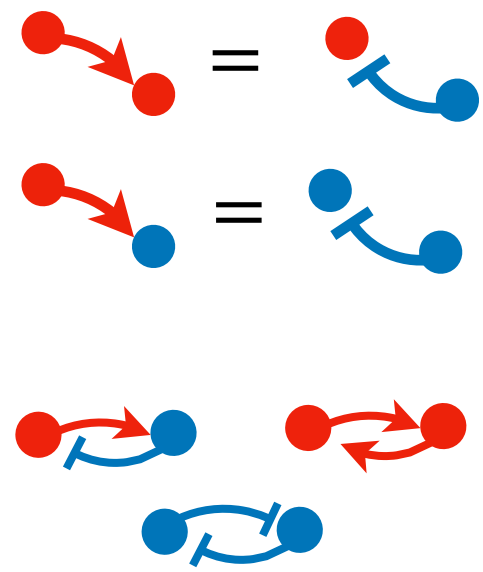
E-I network, reciprocal motifs

Motifs **First-order**



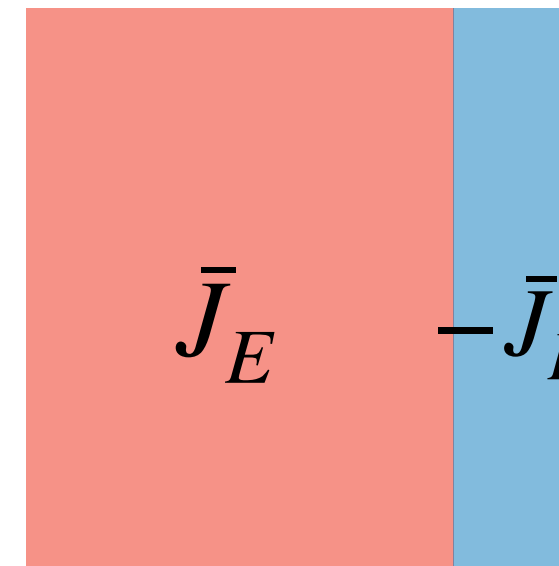
E-I network, reciprocal motifs

Motifs **First-order**



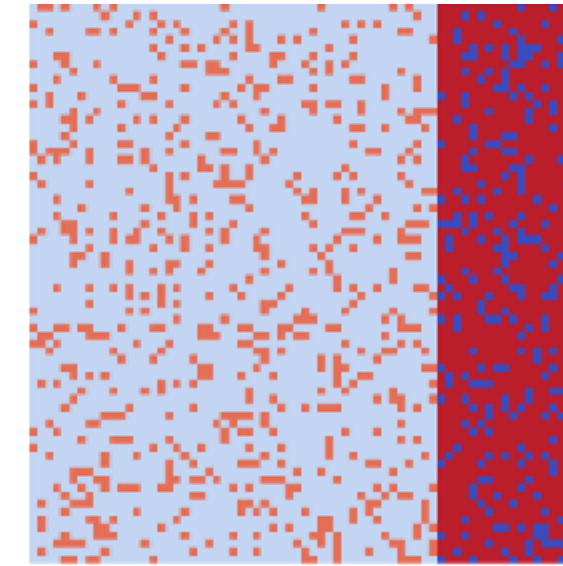
=

Mean



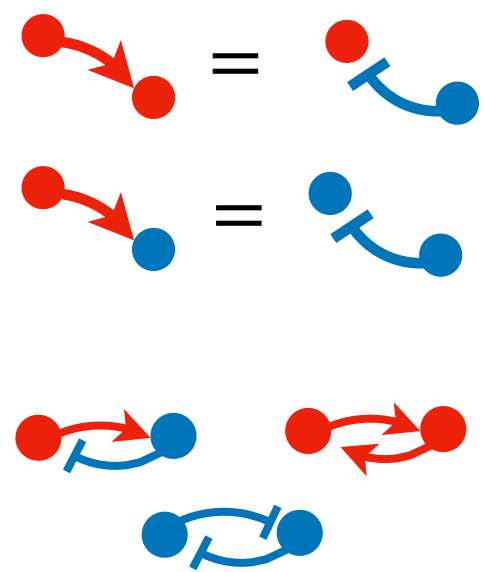
+

Fluctuations

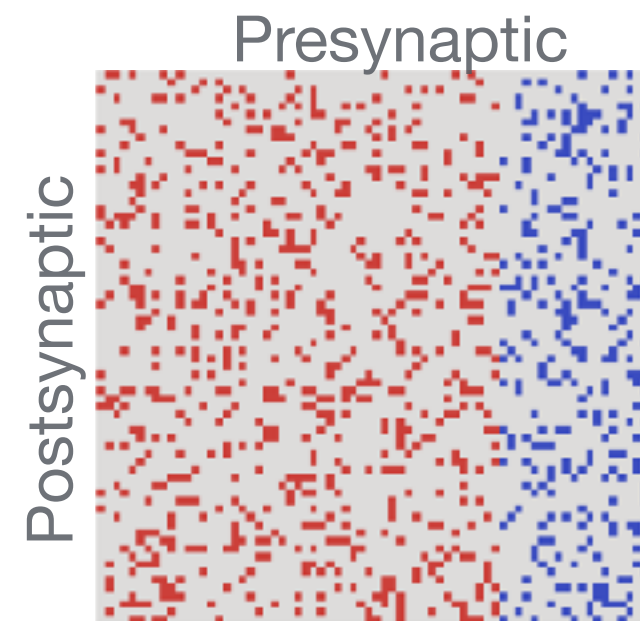


E-I network, reciprocal motifs

First-order
Motifs

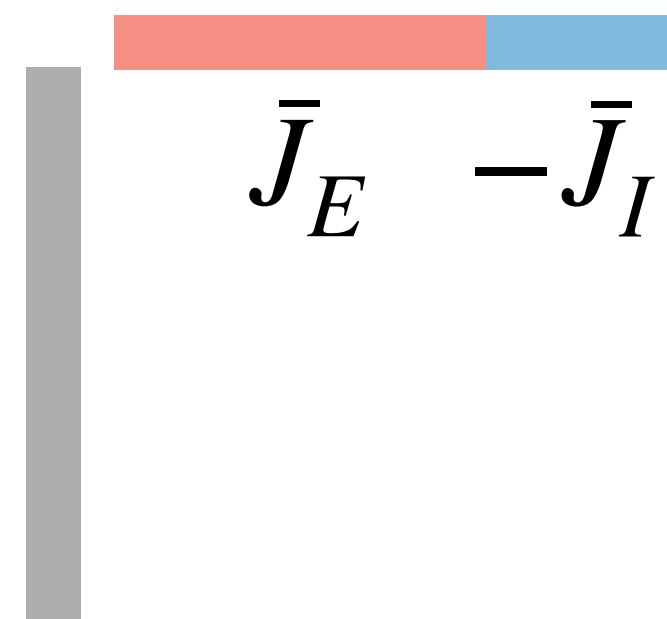


Reciprocal
(Symmetric)



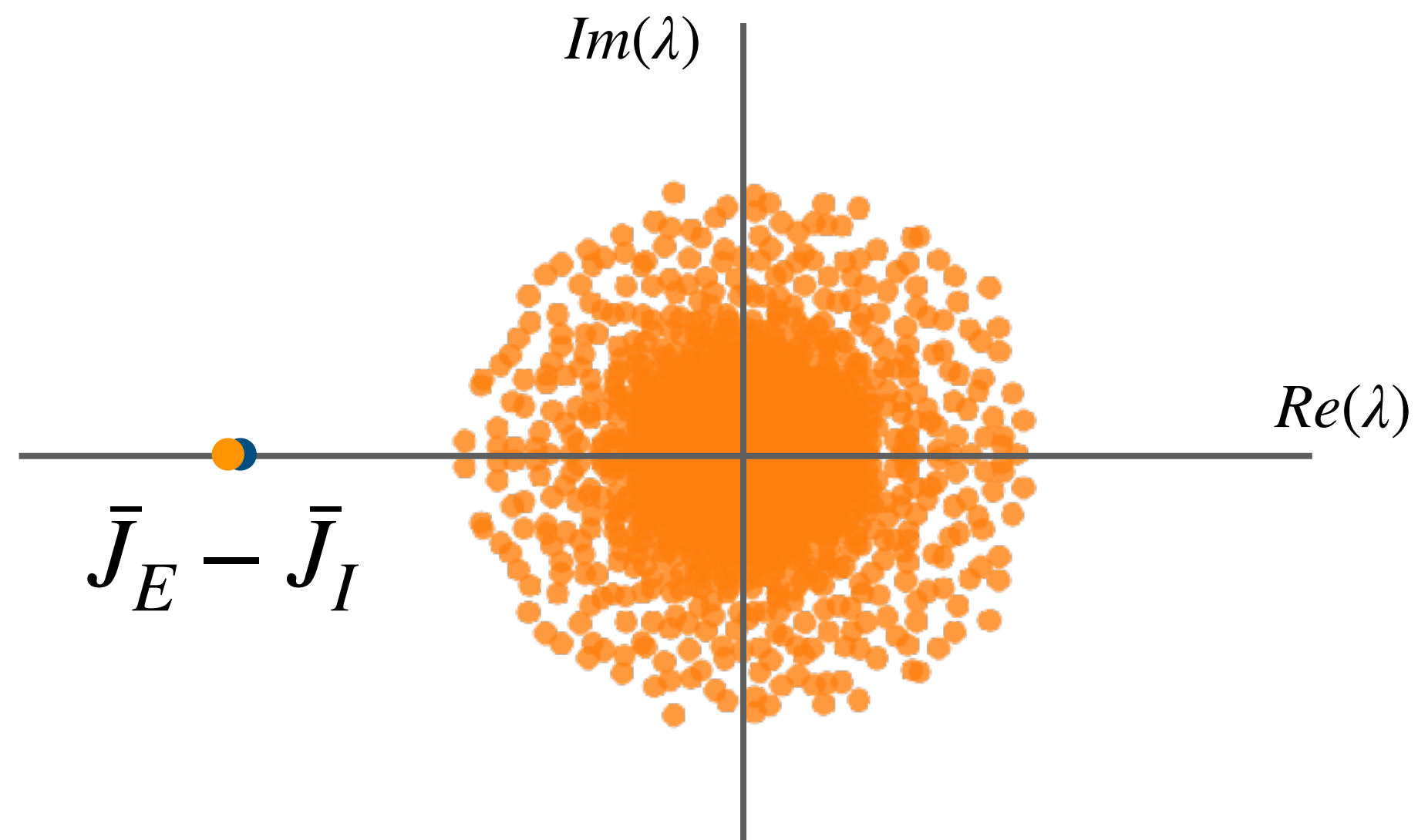
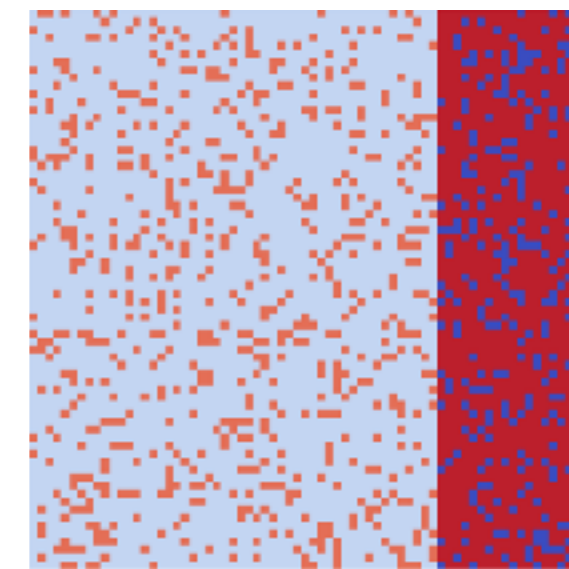
=

Unit Rank



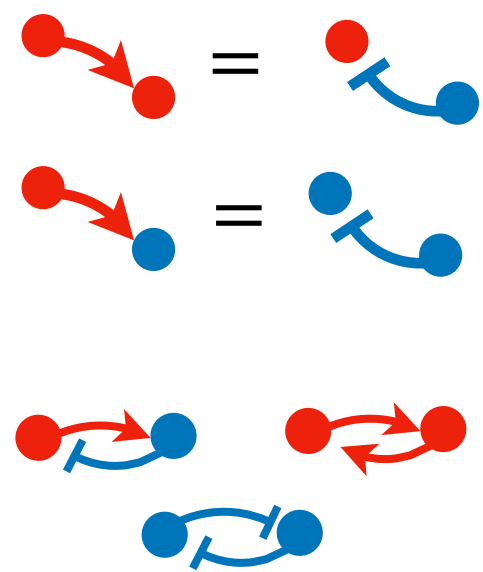
+

Random **Correlated**

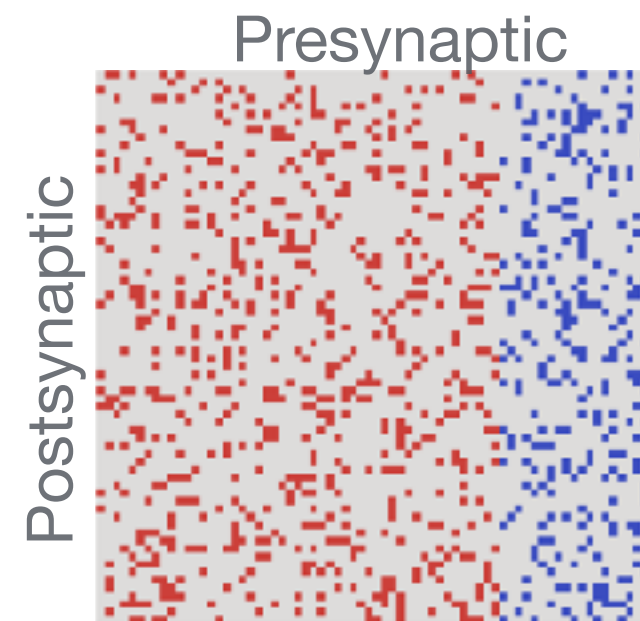


E-I network, reciprocal motifs

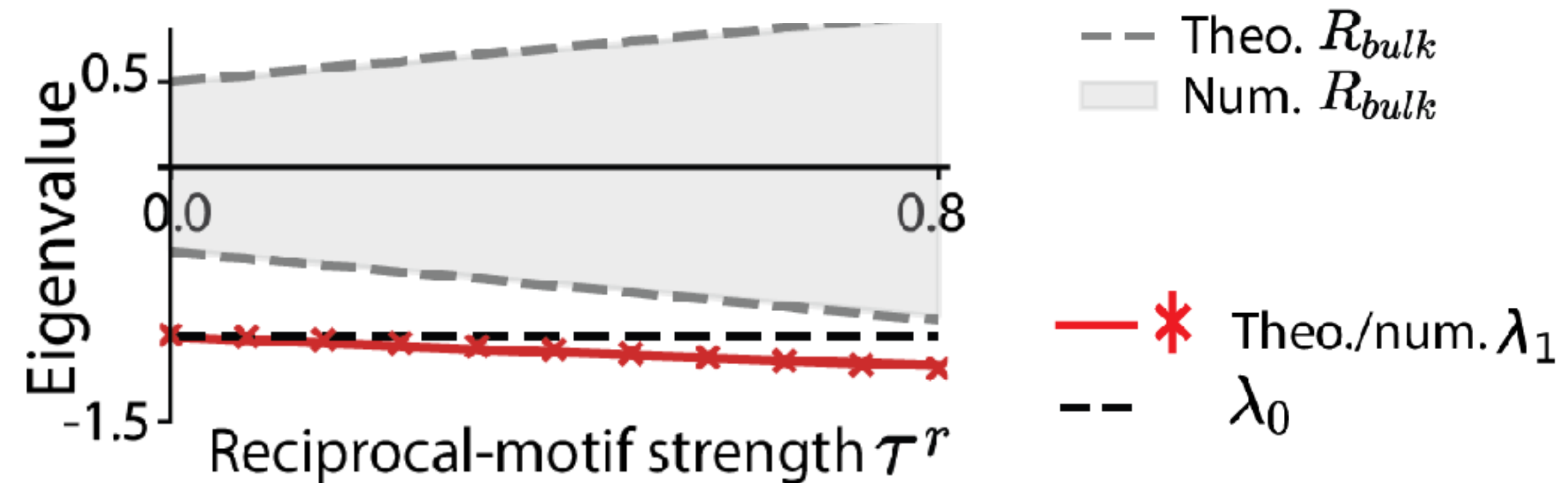
First-order
Motifs



Reciprocal
(Symmetric)

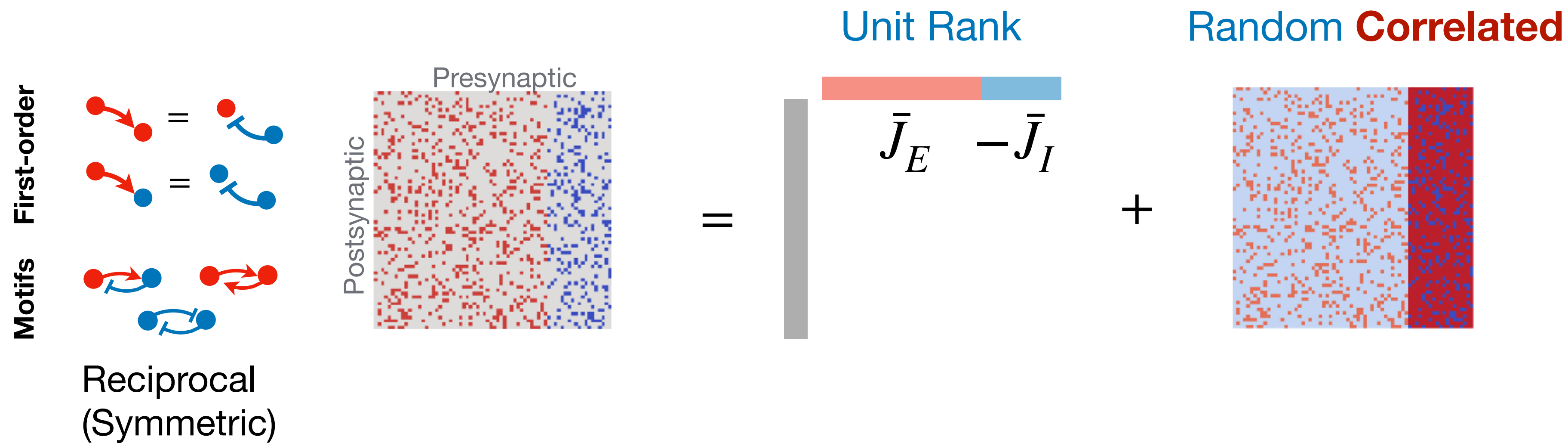


$$= \begin{matrix} \text{Unit Rank} \\ \bar{J}_E & -\bar{J}_I \end{matrix} + \begin{matrix} \text{Random} & \text{Correlated} \end{matrix}$$



Minor correction to average connectivity

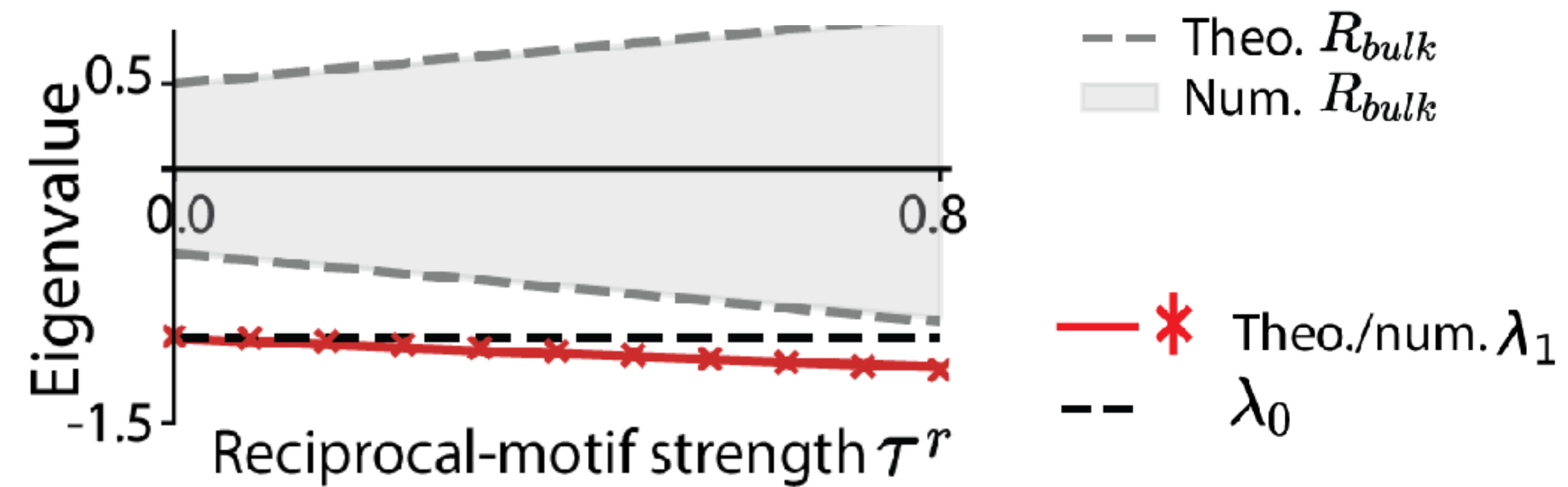
E-I network, reciprocal motifs



$$[\theta_k] = \frac{1}{N} \bar{\mathbf{n}}^T [\mathbf{Z}^k] \bar{\mathbf{m}} = 0$$

$$f(\lambda) = \lambda^3 - (\lambda_0 \lambda^2 + \theta_1 \lambda + \theta_2) = 0$$

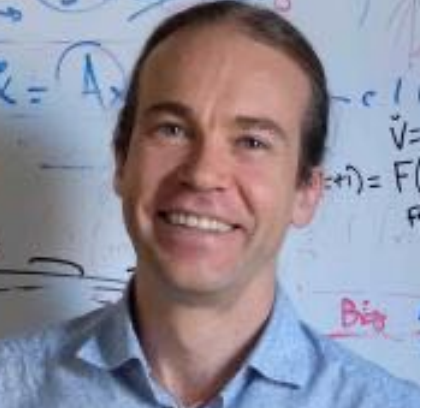
$$[\theta_2] = \frac{1}{N} \bar{\mathbf{n}}^T [\mathbf{Z}^2] \bar{\mathbf{m}} \neq 0$$



Minor correction to average connectivity



Stefano Recanatesi David Dahmen

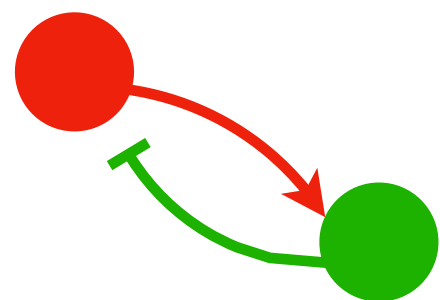


Eric Shea-Brown

Local Connectivity

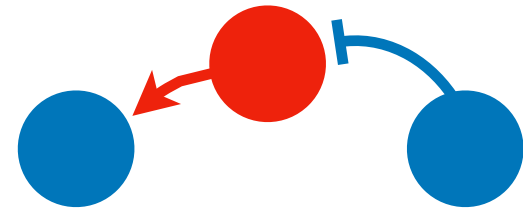
Correlations = *Connectivity motifs*

reciprocal



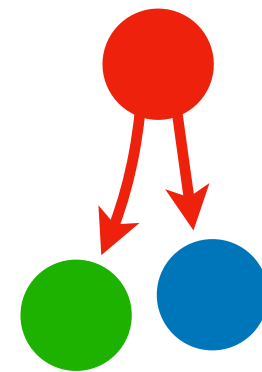
τ^r

chain



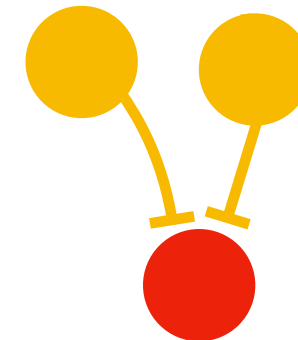
τ^c

divergent

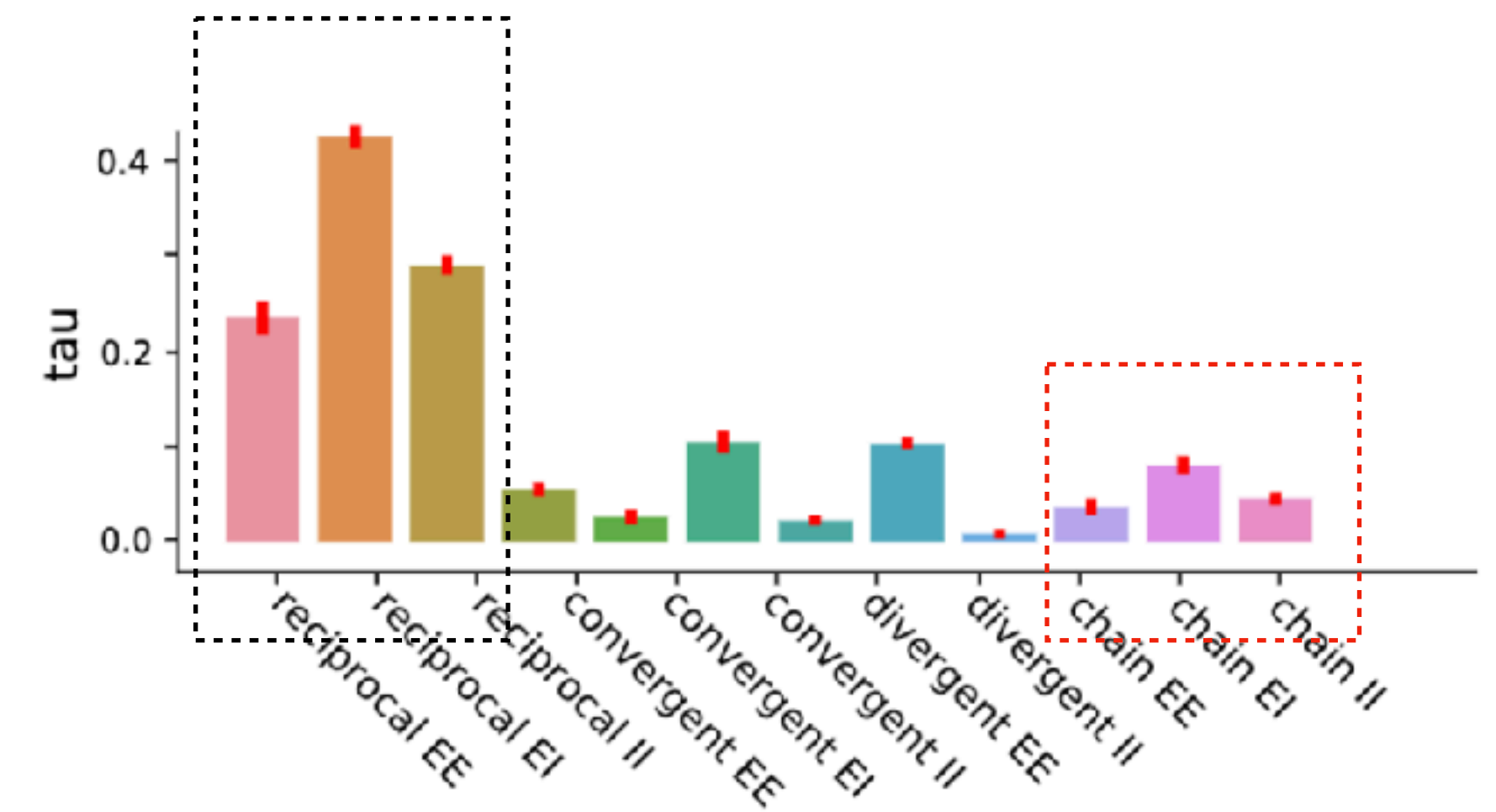


τ^{div}

convergent



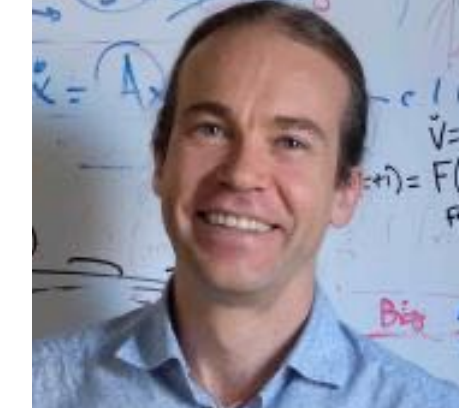
τ^{con}



[Recanatesi et al 2020]



Stefano Recanatesi David Dahmen

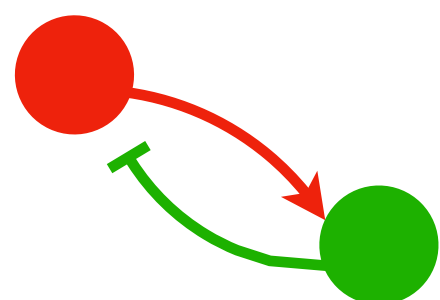


Eric Shea-Brown

Local Connectivity

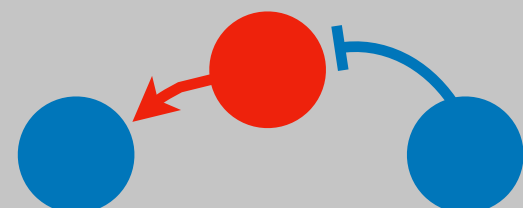
Correlations = *Connectivity motifs*

reciprocal



τ^r

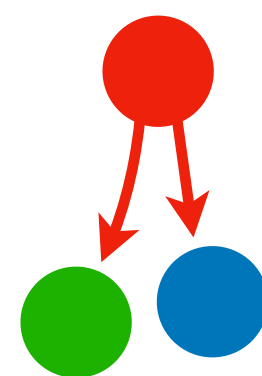
chain



τ^c

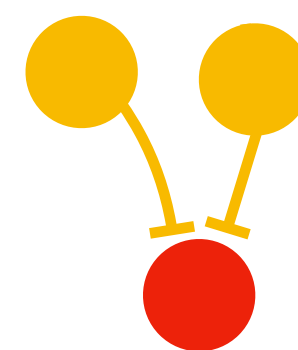
[Hu et al 2018]

divergent

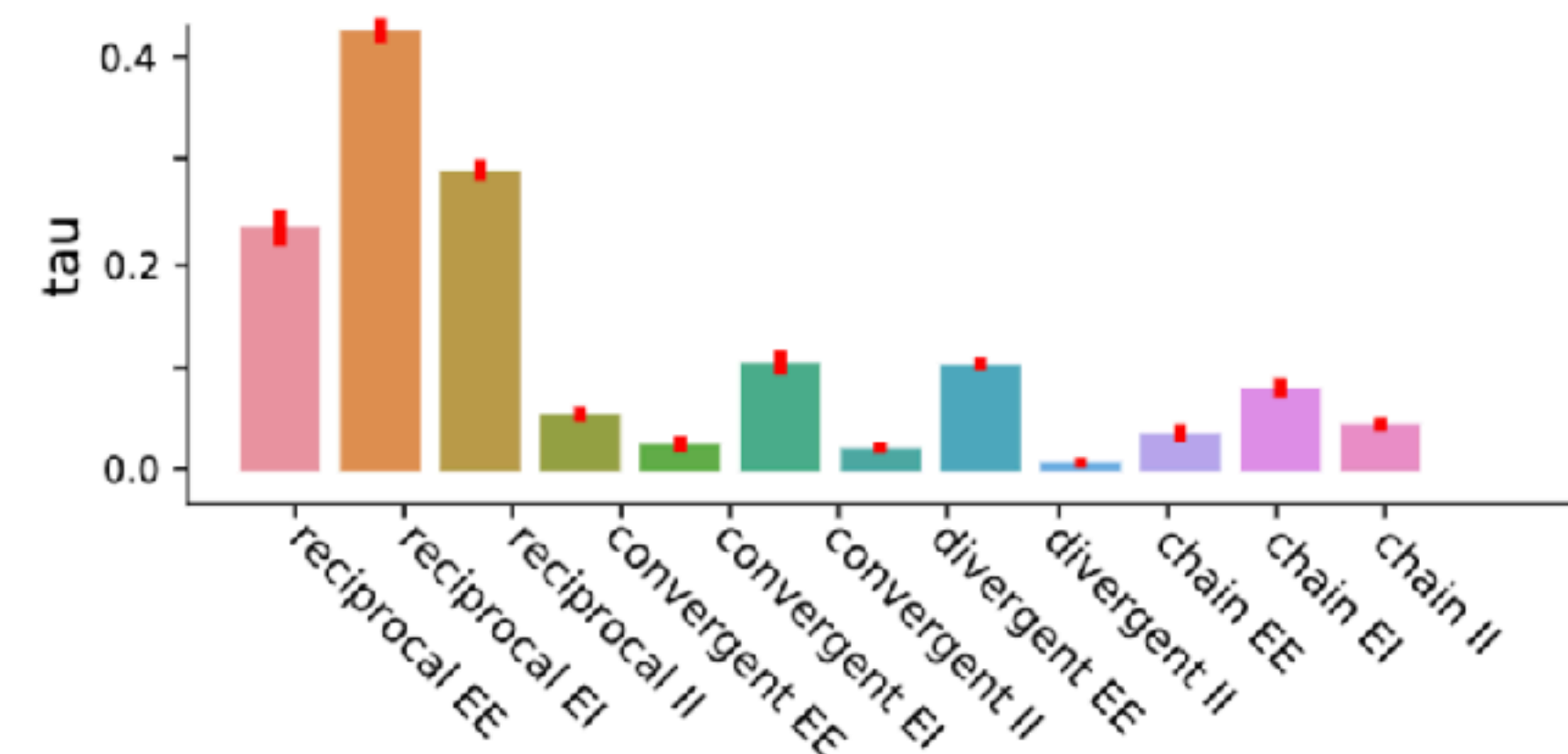


τ^{div}

convergent

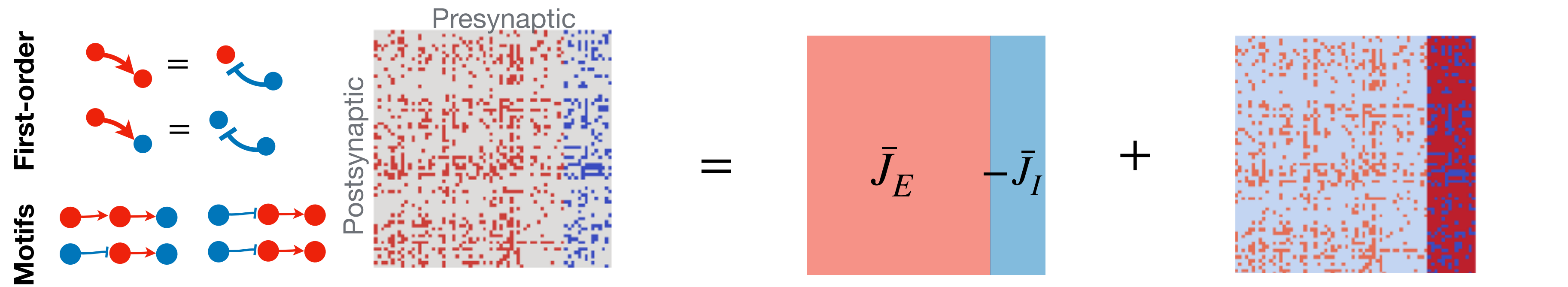


τ^{con}

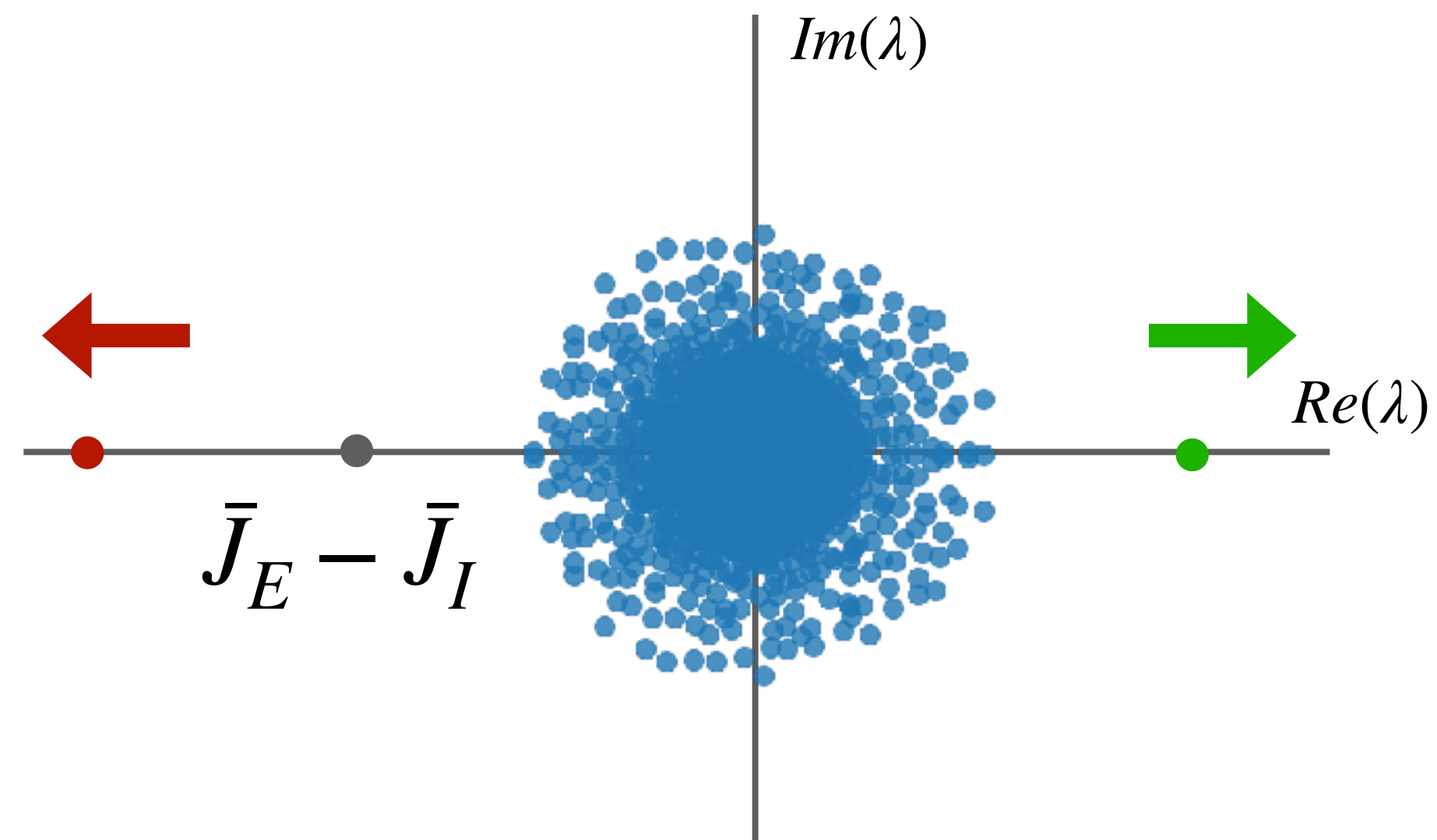
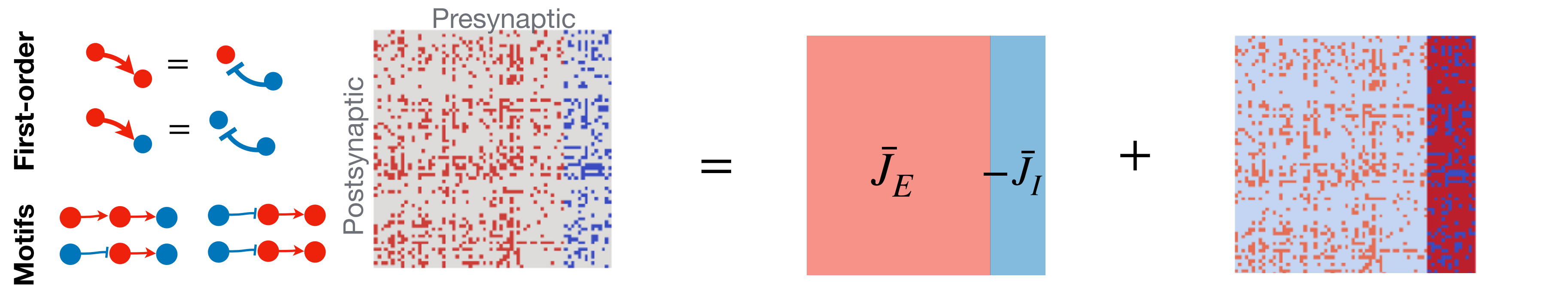


[Recanatesi et al 2020]

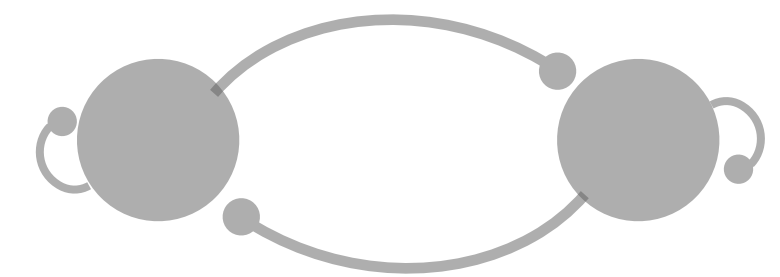
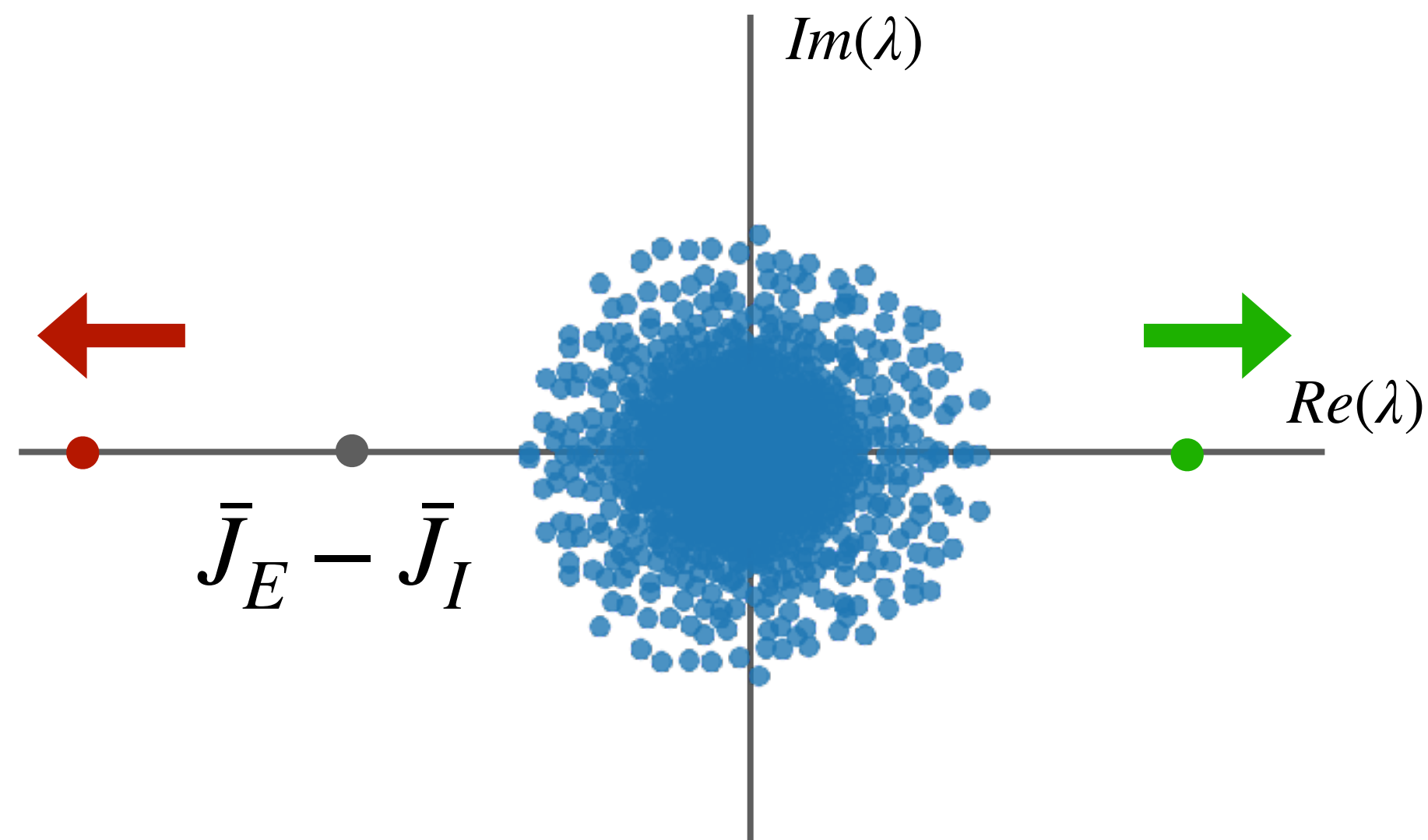
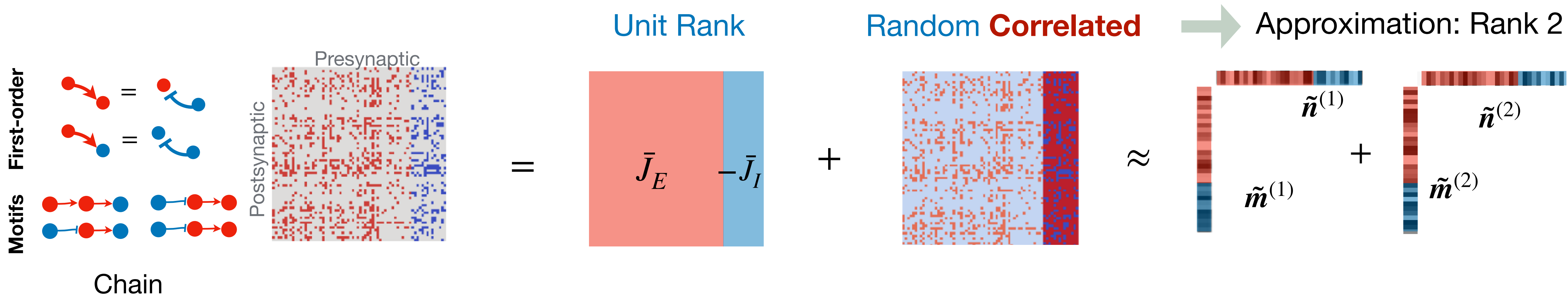
E-I network, chain motifs



E-I network, chain motifs

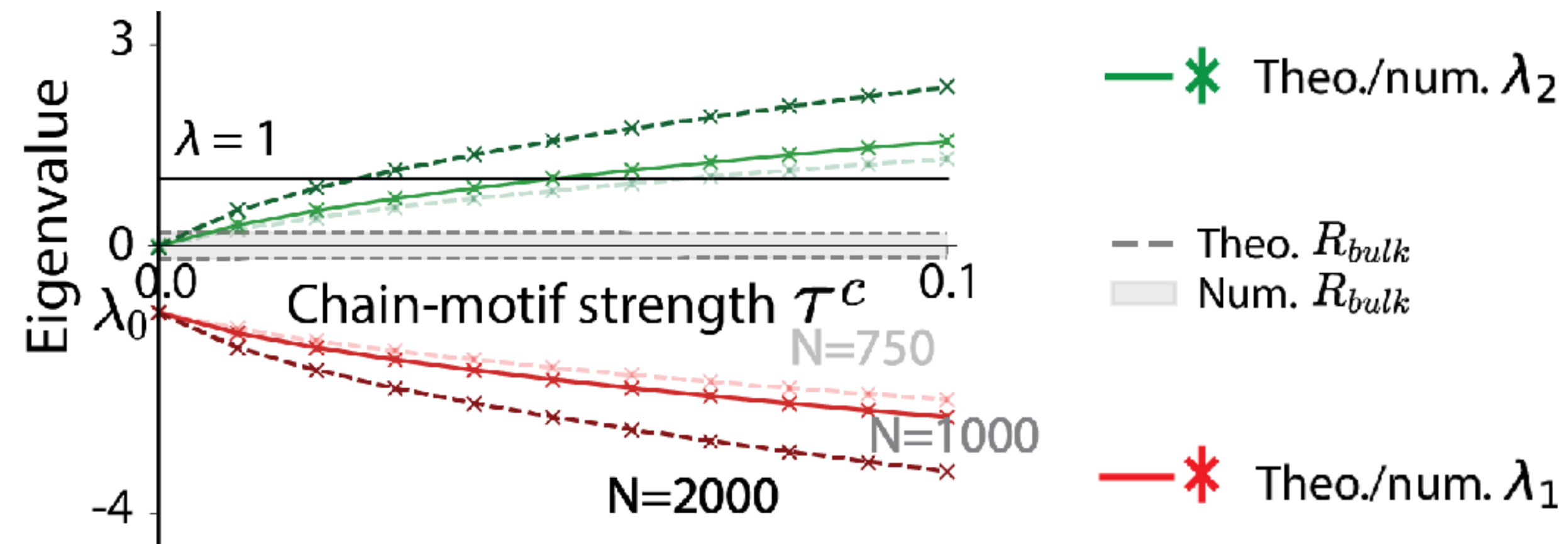
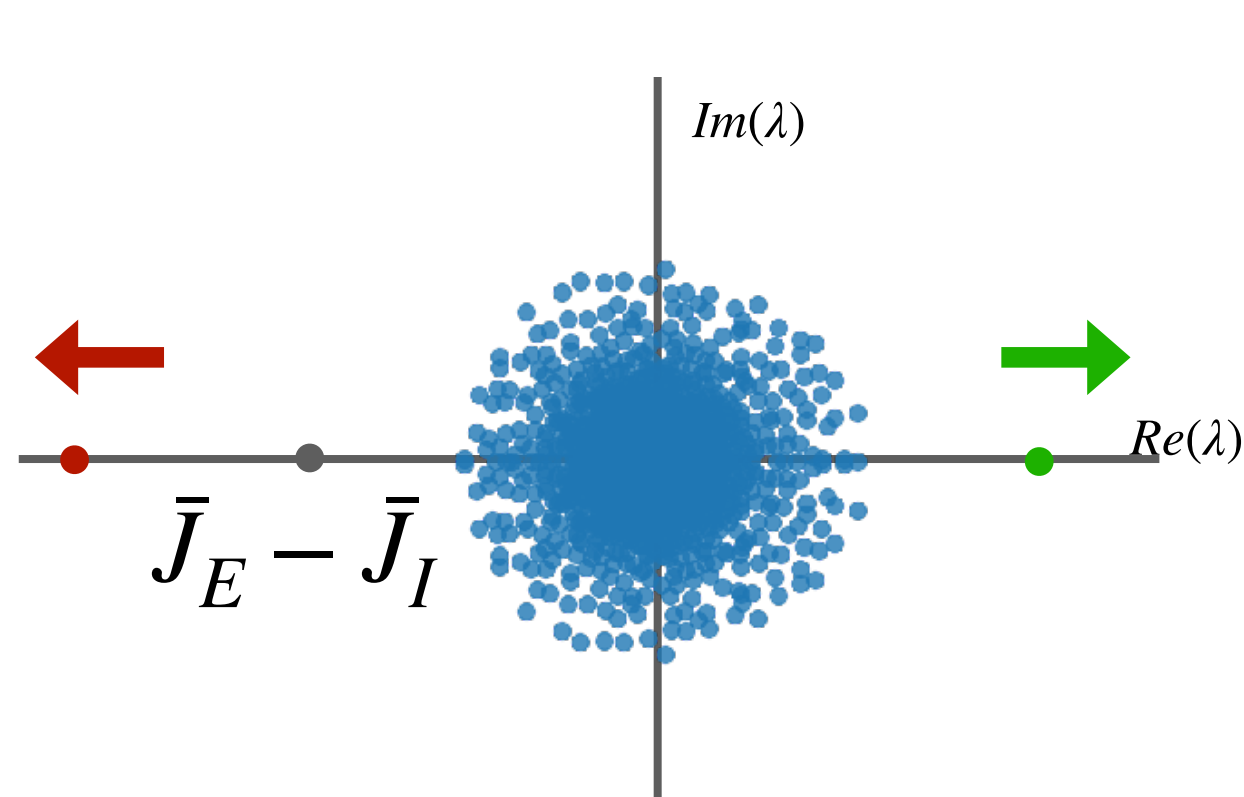
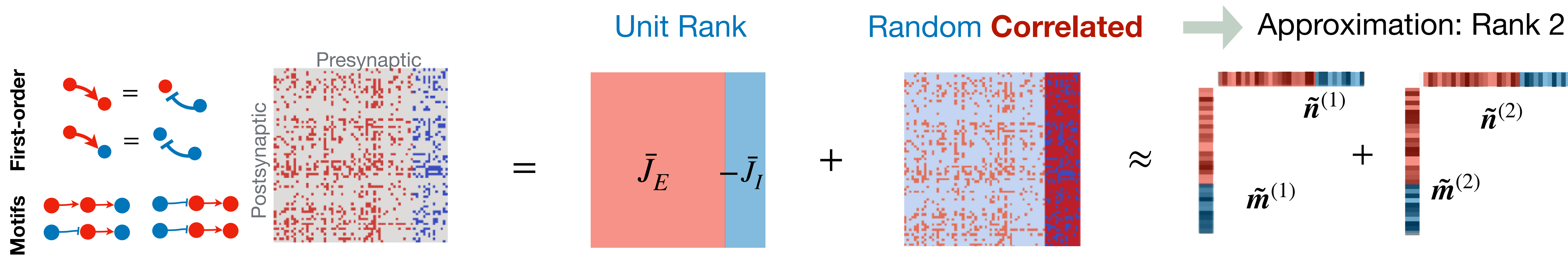


E-I network, chain motifs



➔ **Strong deviations from predictions of mean connectivity**

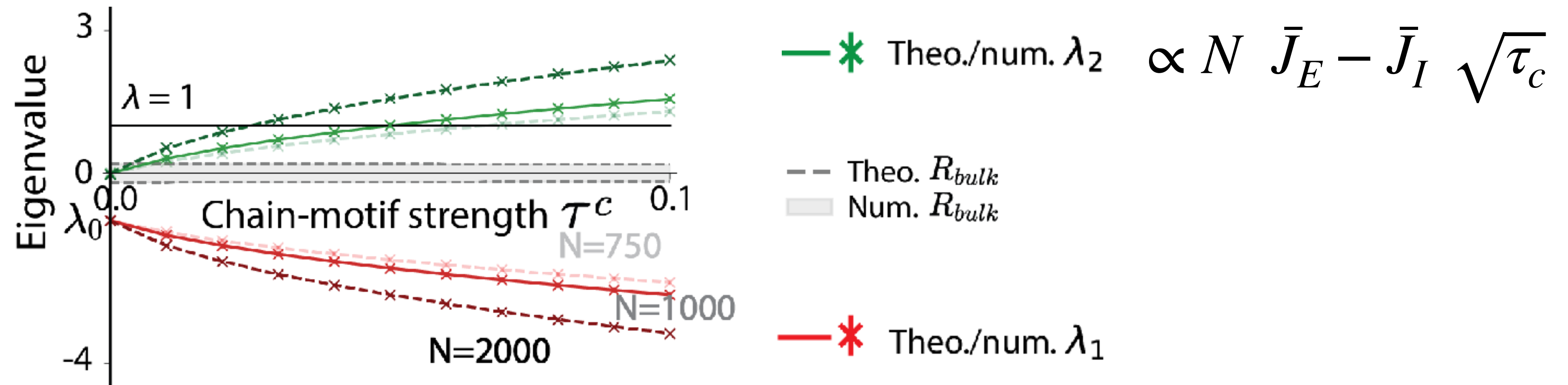
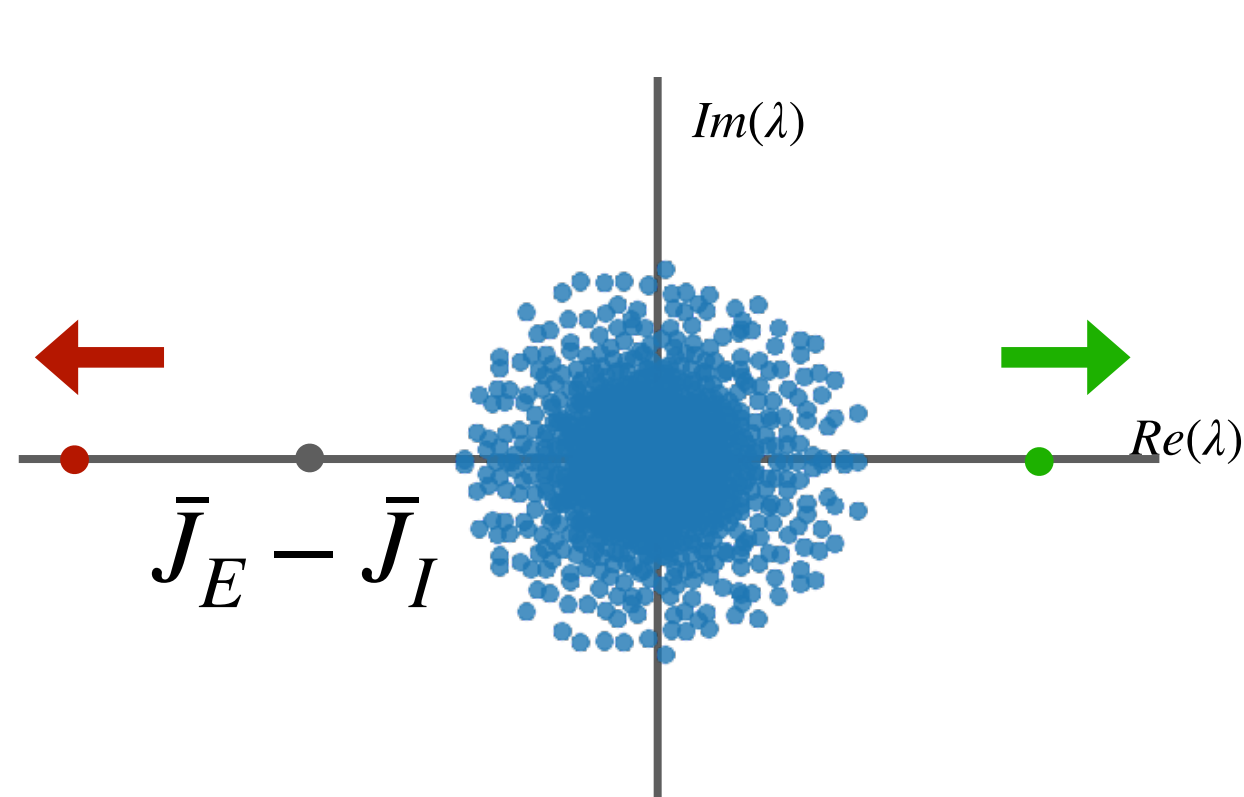
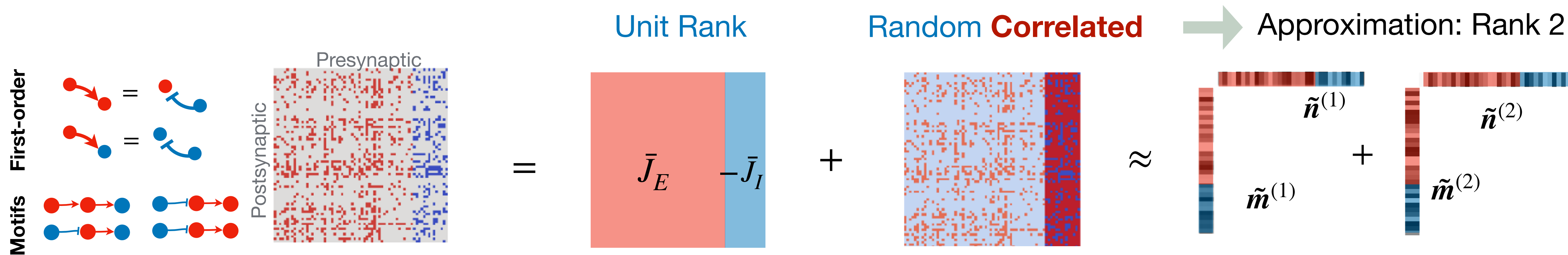
E-I network, chain motifs



[Shao et al 2024]

➔ Potential instability in inhibition-dominated networks

E-I network, chain motifs



[Shao et al 2024]

➔ Potential instability in inhibition-dominated networks

Stabilizing E-I networks with chain motifs

$$N \bar{J}_E - \bar{J}_I \sqrt{\tau_c} \sim O(1)$$

Weakly Connected Regime

[Brunel 2000]

$$\bar{J}_E, \bar{J}_I \sim O\left(\frac{1}{N}\right)$$

$$\tau_c \sim O(1)$$

Loose balance

Strongly Connected Regime

[Renart et al 2010]

$$\bar{J}_E, \bar{J}_I \sim O\left(\frac{1}{\sqrt{N}}\right)$$

$$\tau_c \sim O(1)$$

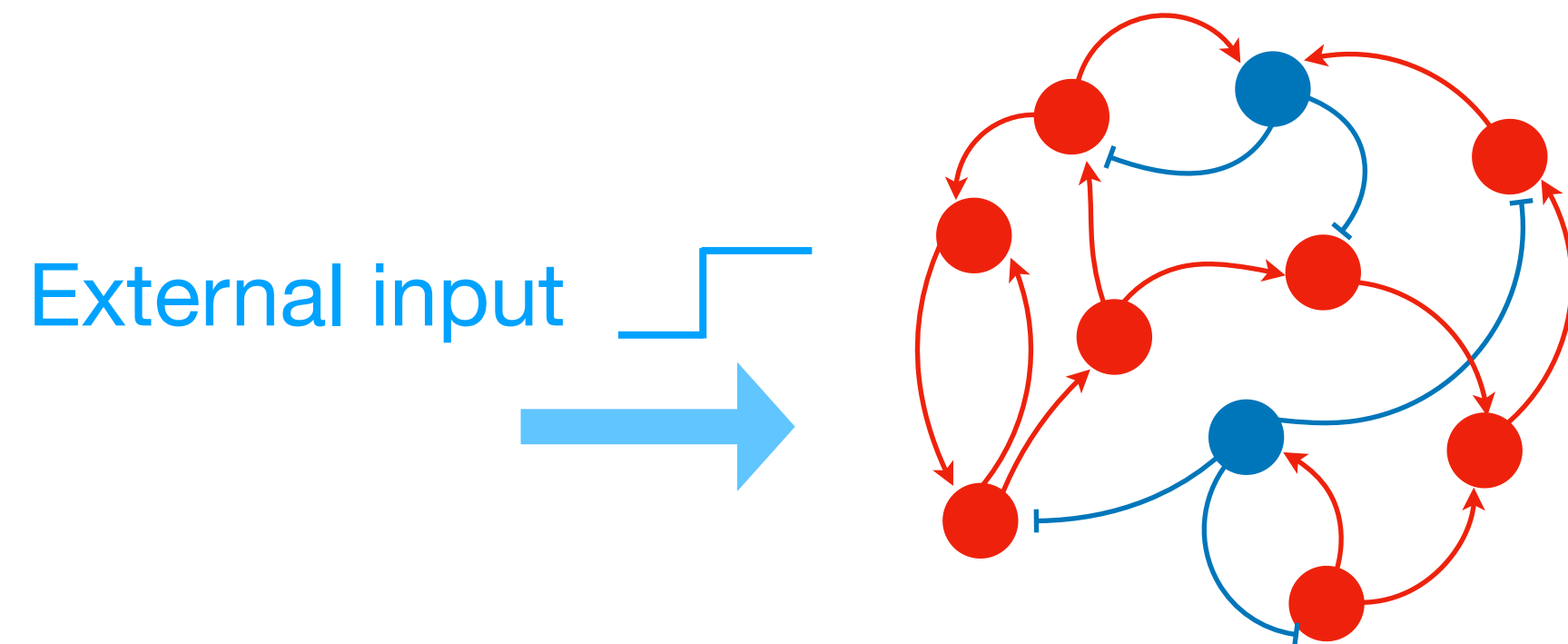

$$\bar{J}_E - \bar{J}_I \sim O\left(\frac{1}{\sqrt{N}}\right)$$

Tight balance

Results

1. Methods: eigenvalues and eigenspectra
2. E-I networks: Eigenvalues and low-rank approximation
3. **Dynamics: (paradoxical) responses to inputs**

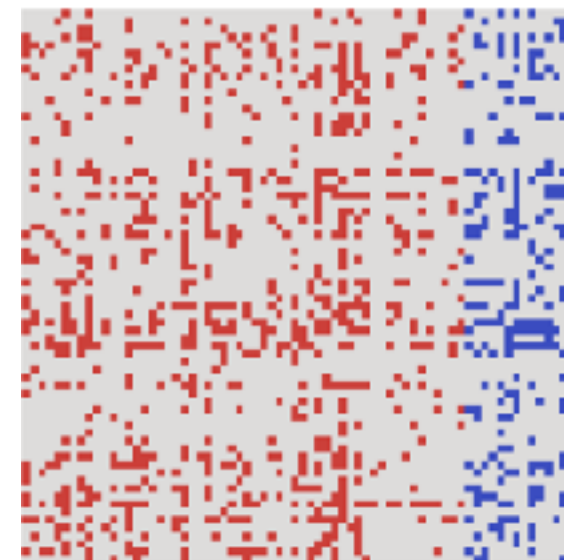
Steady-state response to inputs



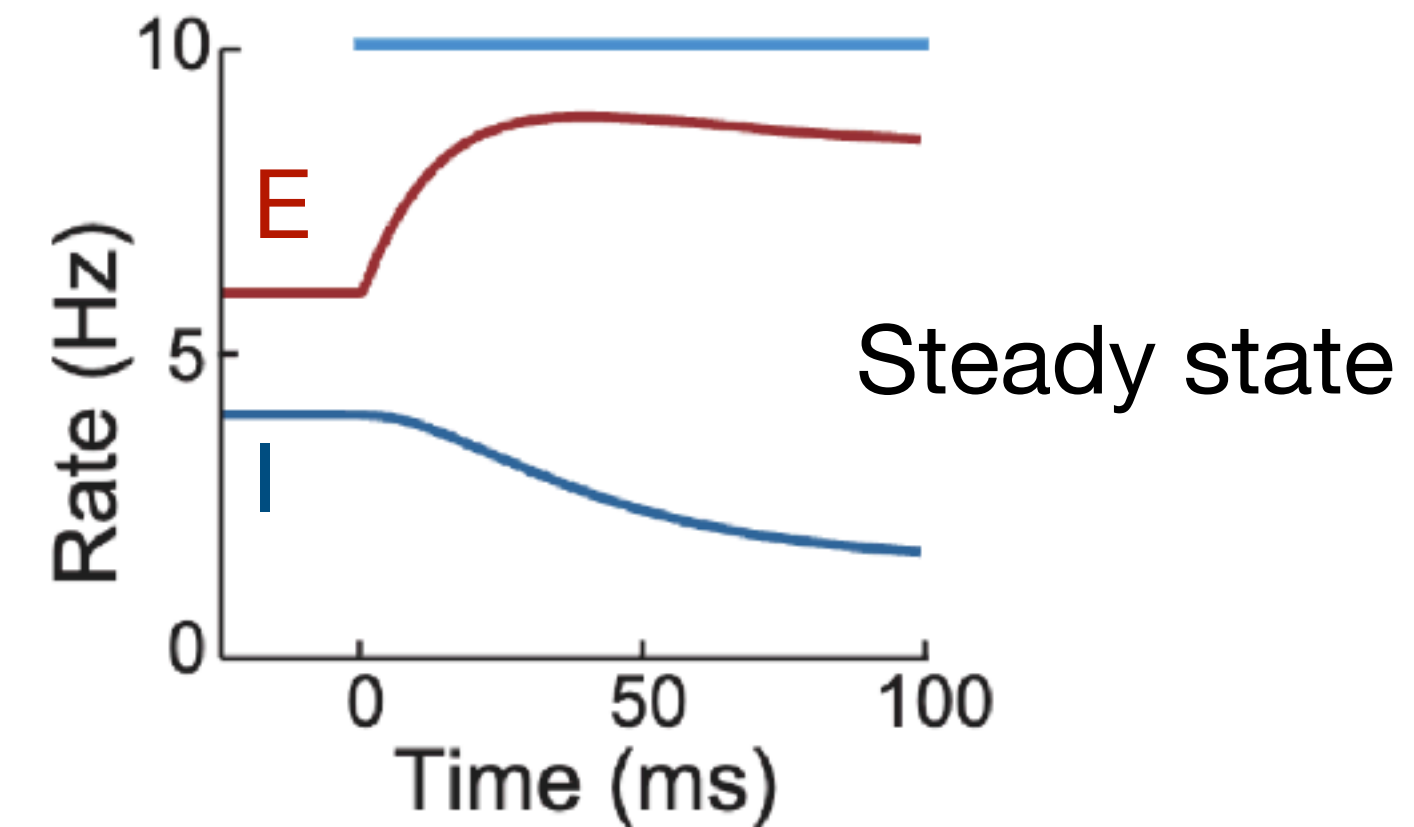
Linear dynamics: $\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{J}\mathbf{r} + \mathbf{I}_{ext}$

Connectivity matrix

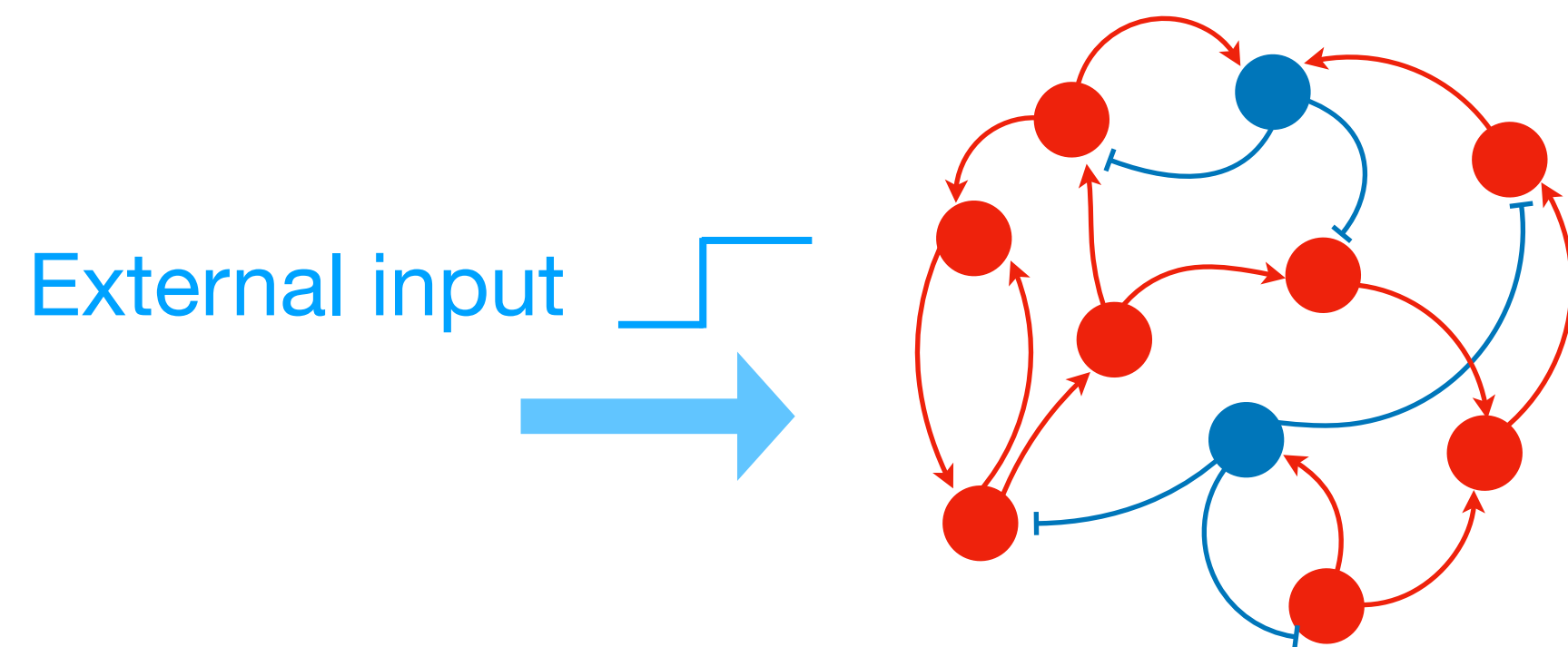
$J =$



Population-averaged activity



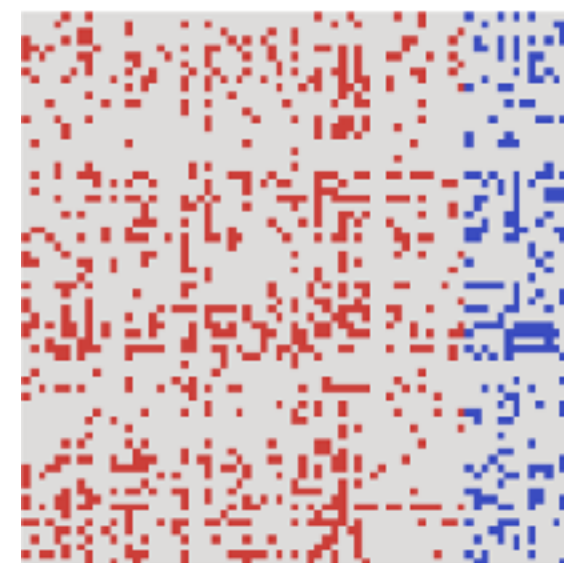
Steady-state response to inputs



Linear dynamics: $\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{J}\mathbf{r} + \mathbf{I}_{ext}$

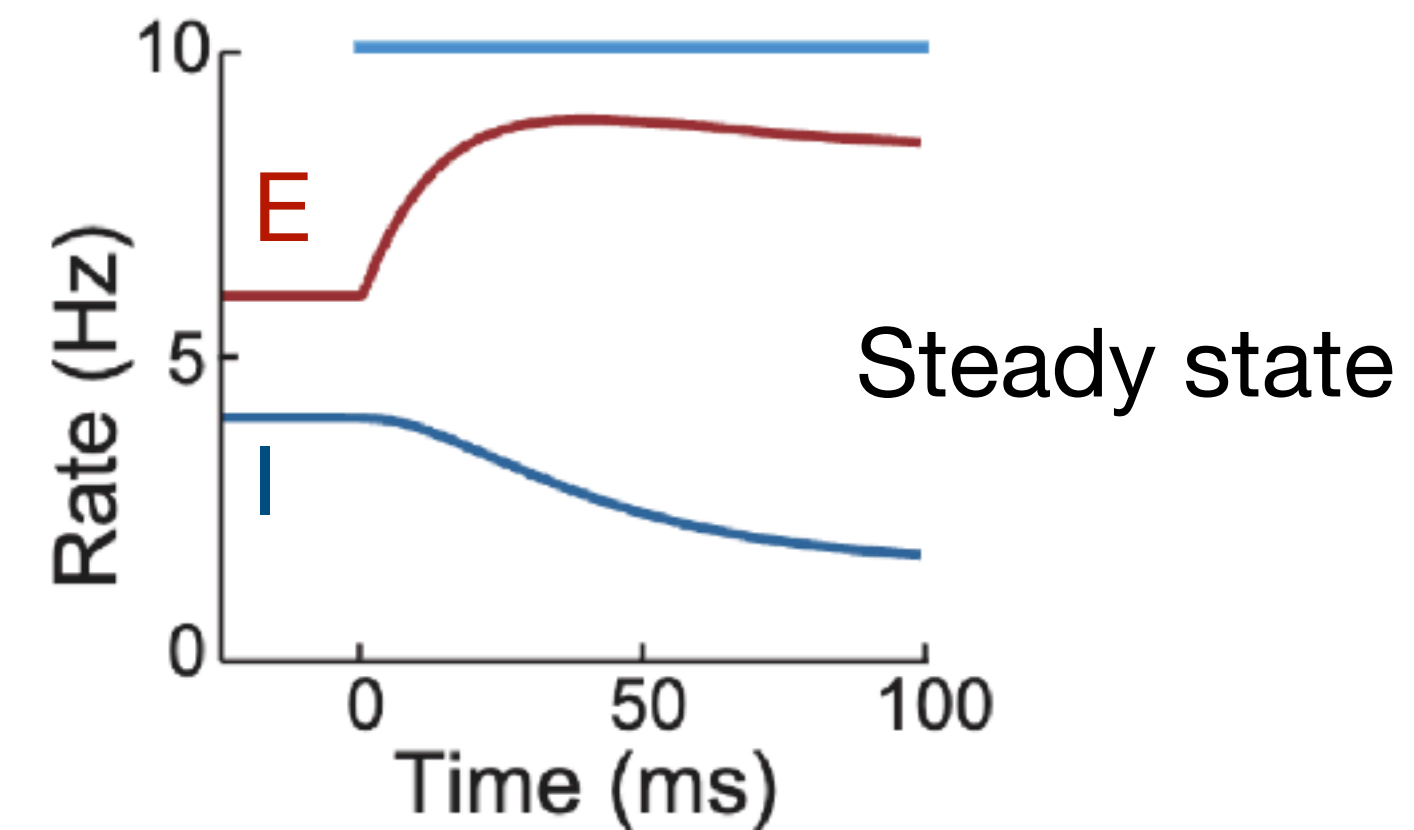
Connectivity matrix

$\mathbf{J} =$

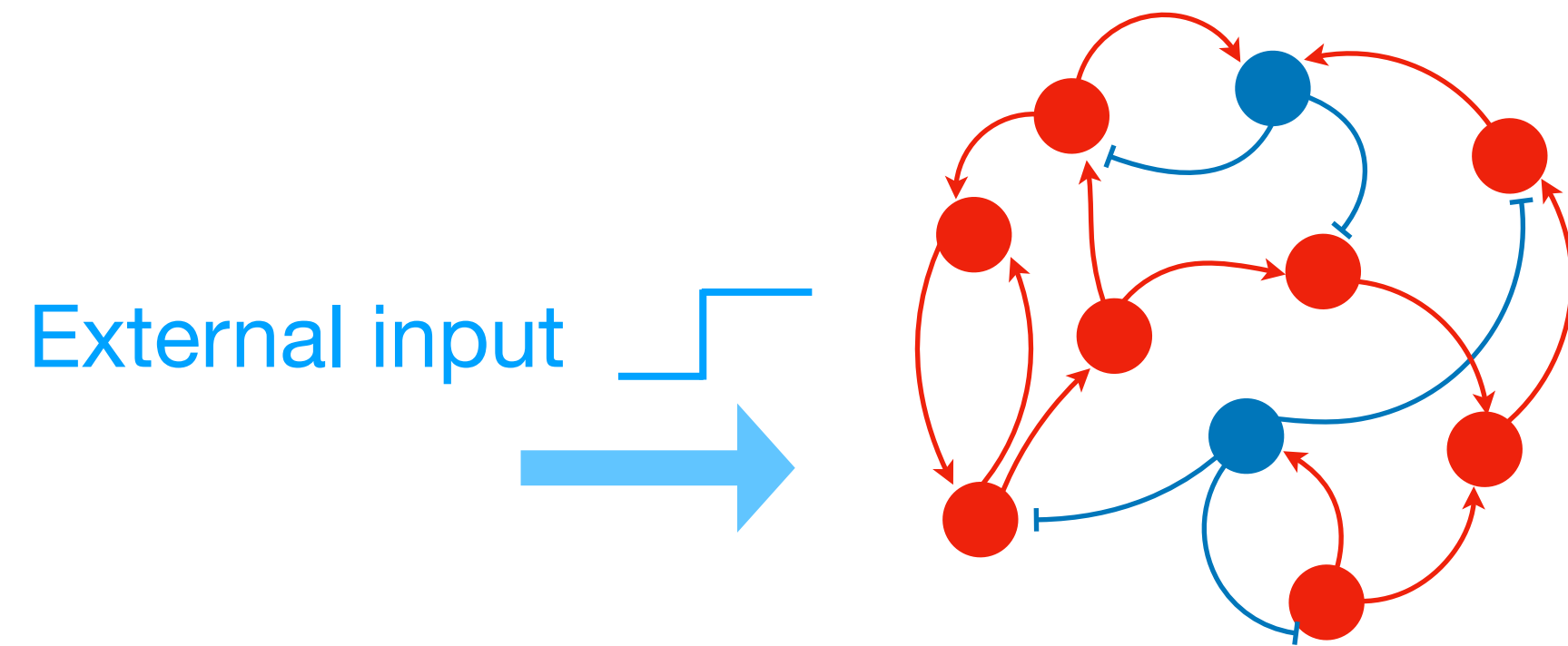


Steady state: $\mathbf{r}^* = (\mathbf{1} - \mathbf{J})^{-1} \mathbf{I}^{ext}$

Population-averaged activity

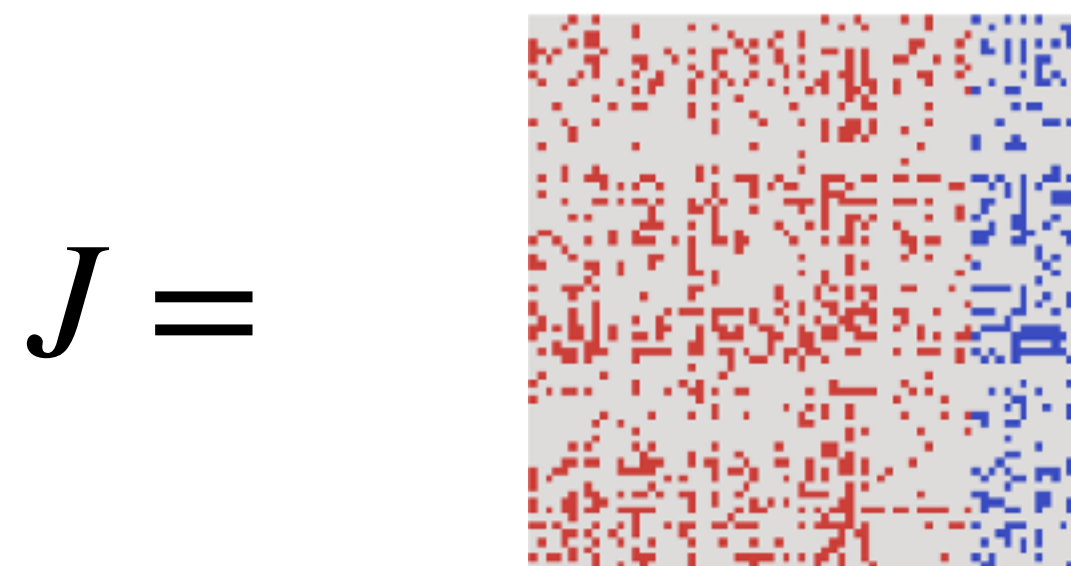


Steady-state response to inputs



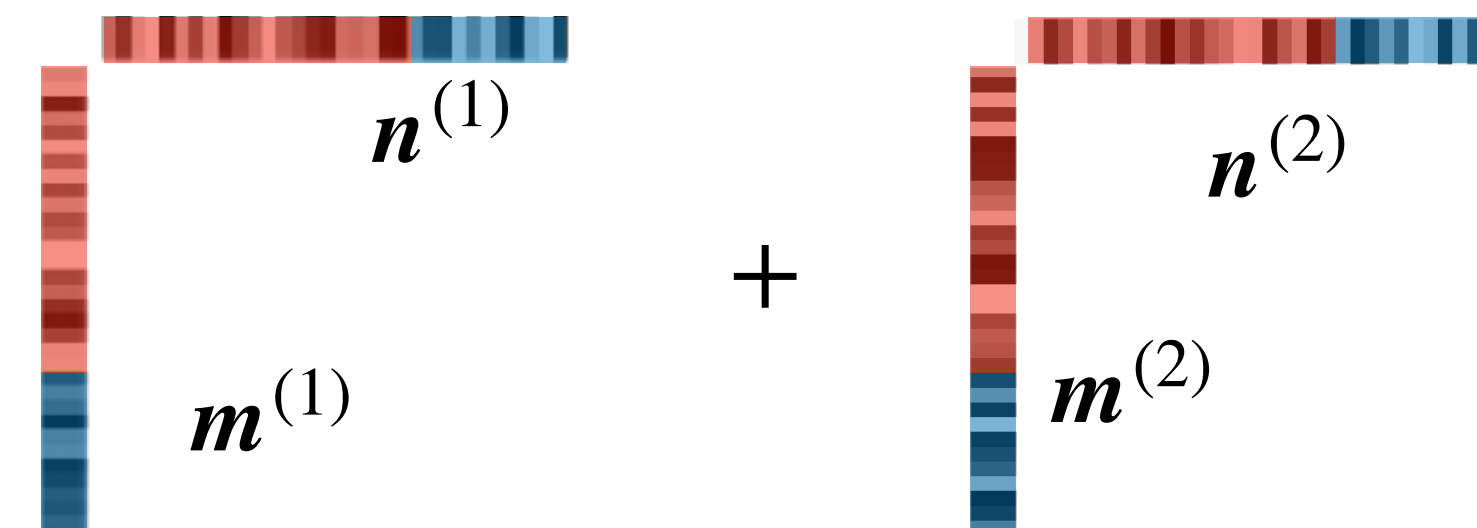
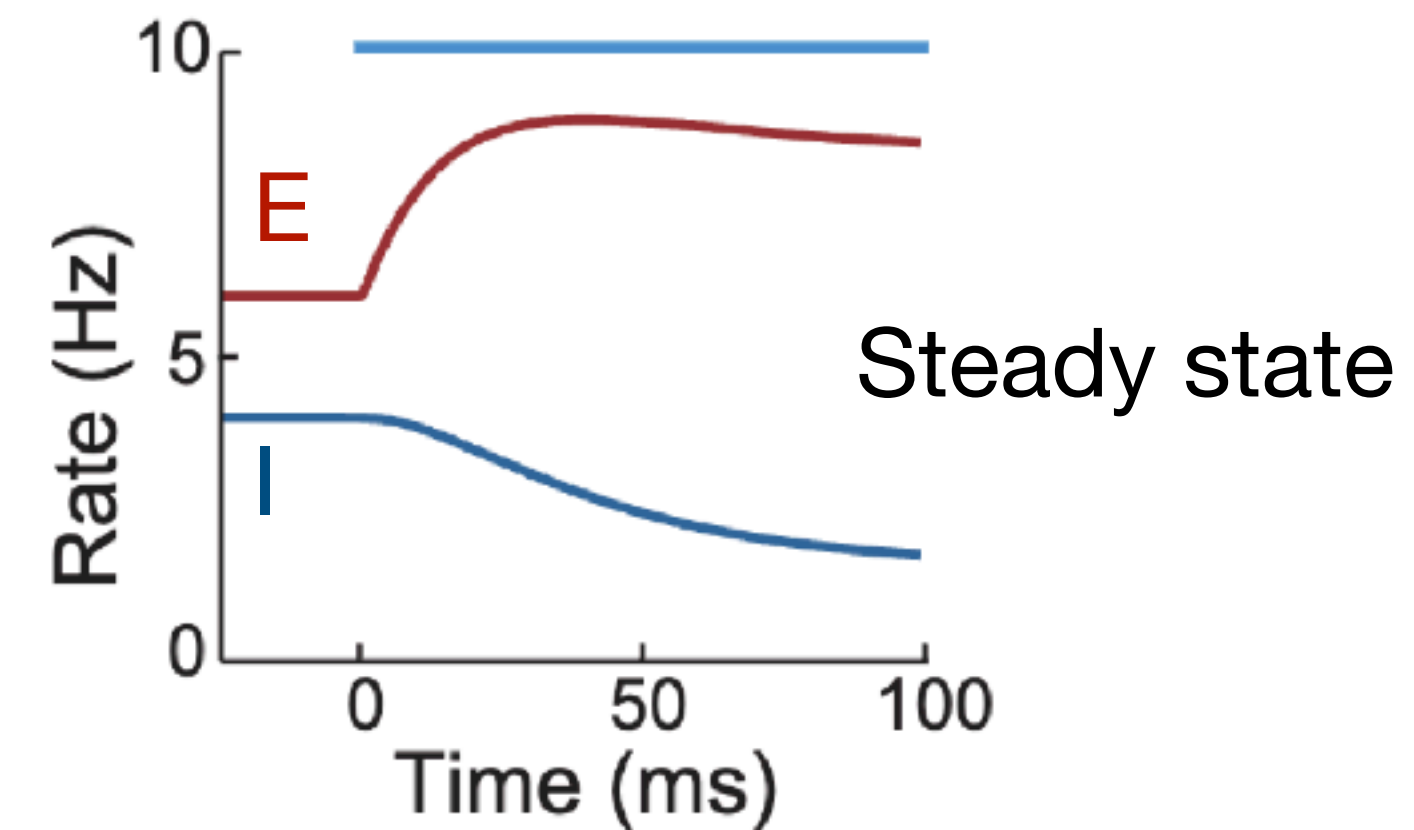
Linear dynamics: $\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{J}\mathbf{r} + \mathbf{I}_{ext}$

Connectivity matrix



$\approx$

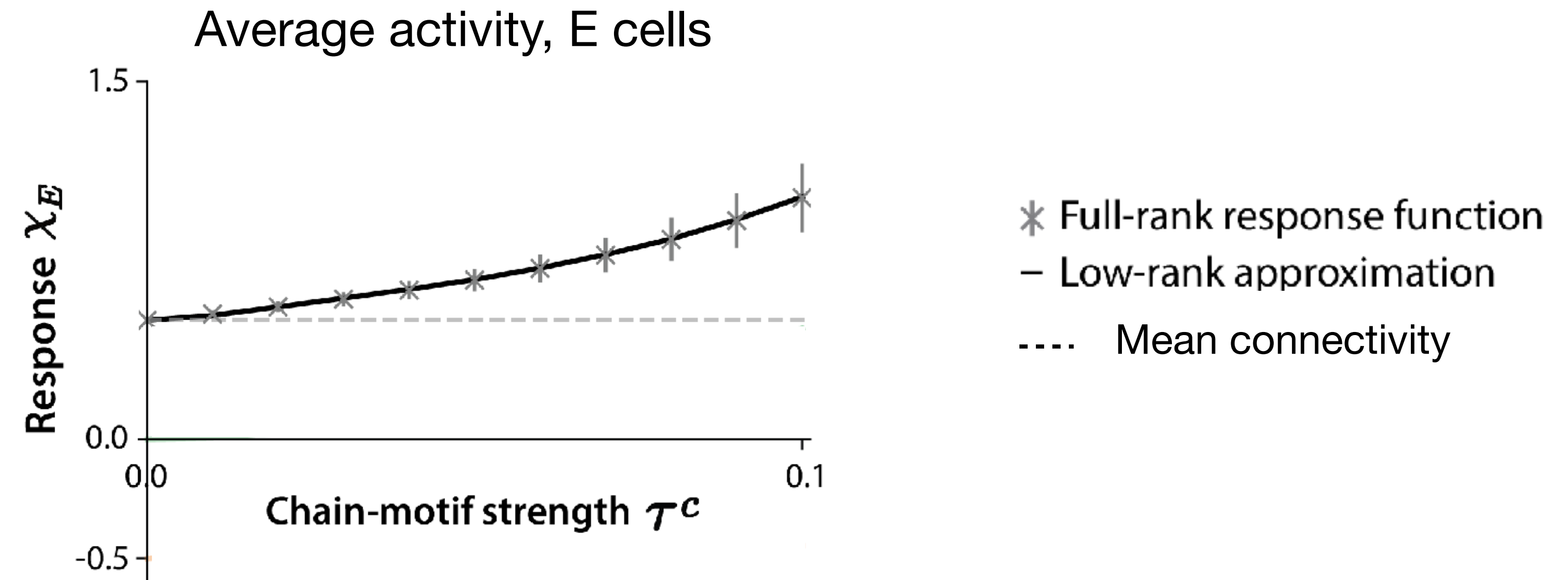
Population-averaged activity



$$\mathbf{r}^* = (\mathbf{1} - \mathbf{J})^{-1} \mathbf{I}^{ext}$$

$$\mathbf{I}^{ext} + \frac{\mathbf{m}^{(1)} \mathbf{n}^{(1)T}}{1 - \lambda_1} \mathbf{I}^{ext} + \frac{\mathbf{m}^{(2)} \mathbf{n}^{(2)T}}{1 - \lambda_2} \mathbf{I}^{ext}$$

Response to uniform input

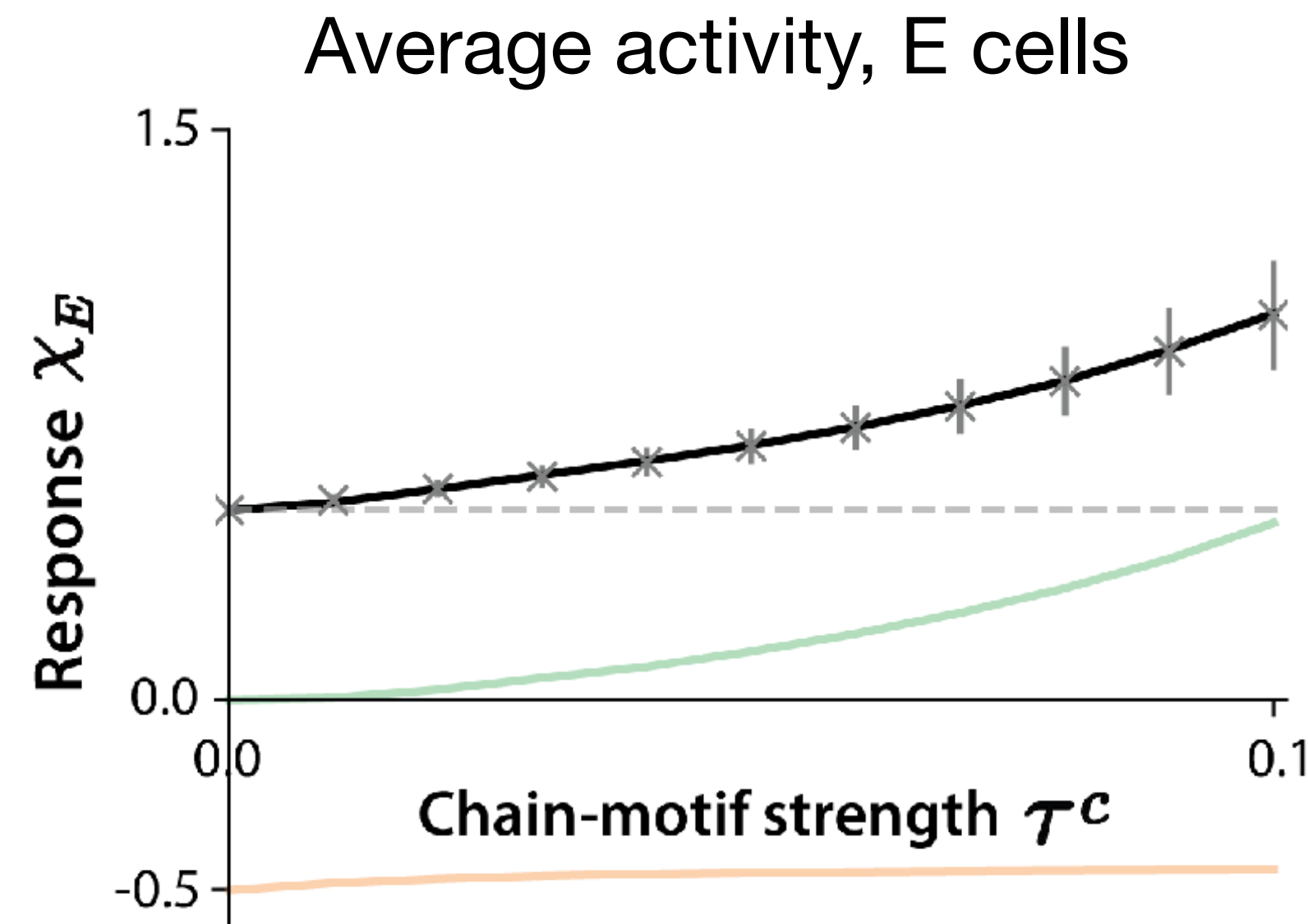
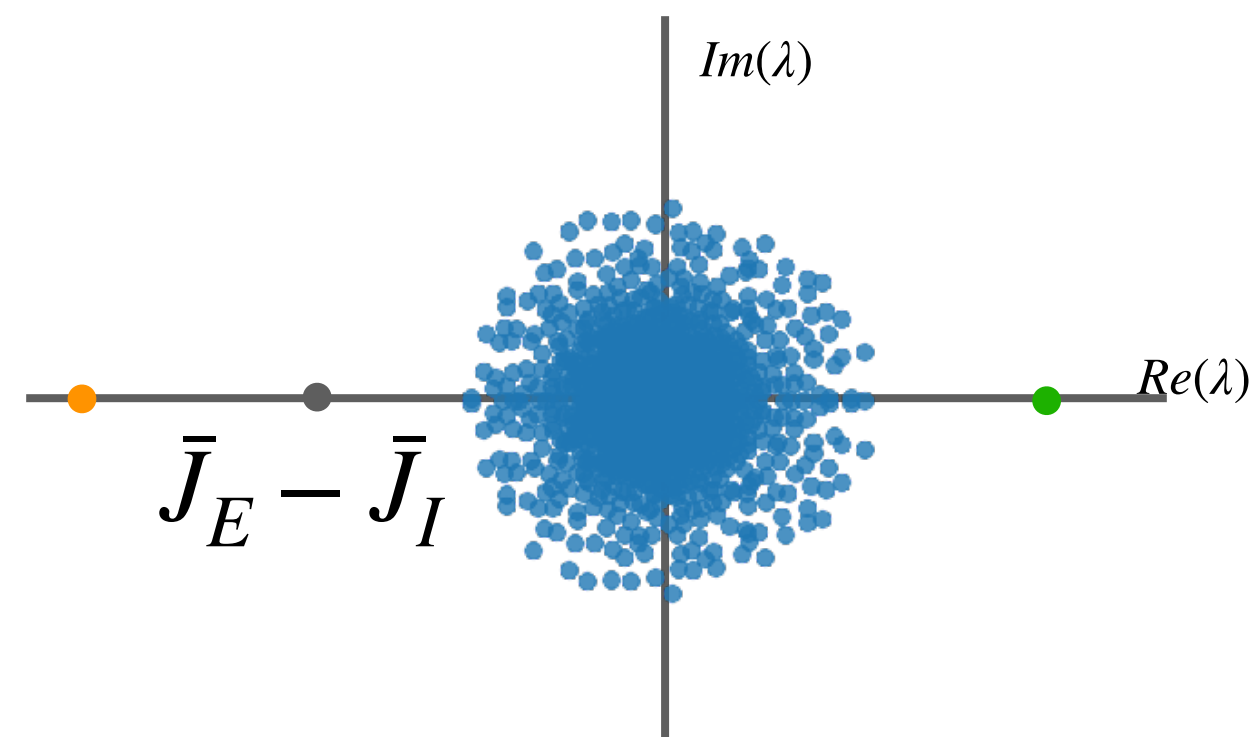


[Shao et al 2024]



Strong deviations from predictions of mean connectivity

Response to uniform input



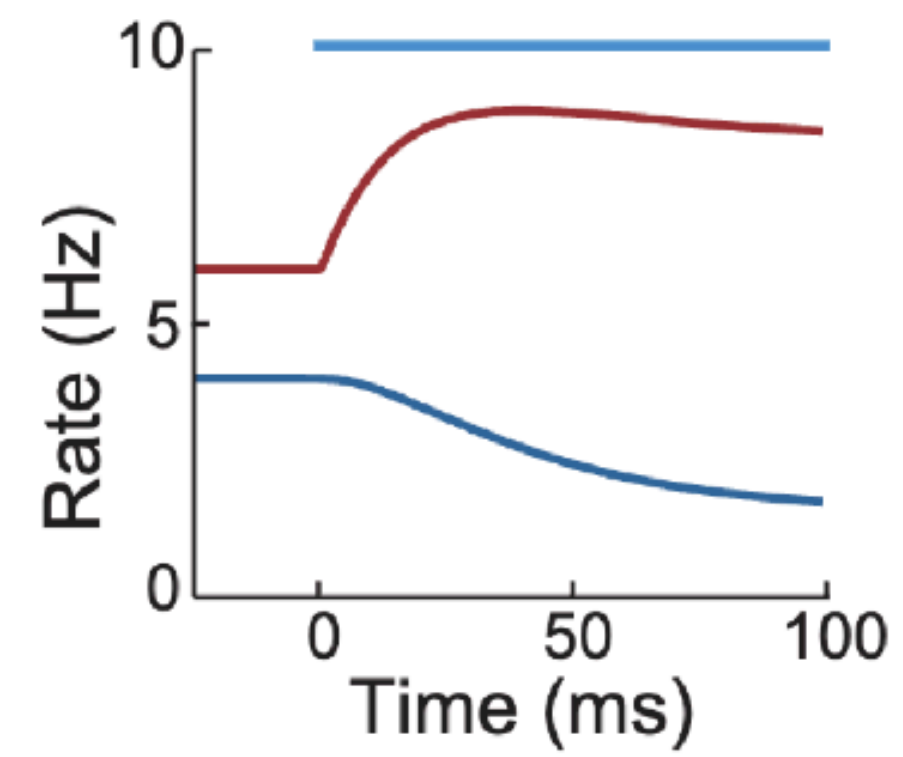
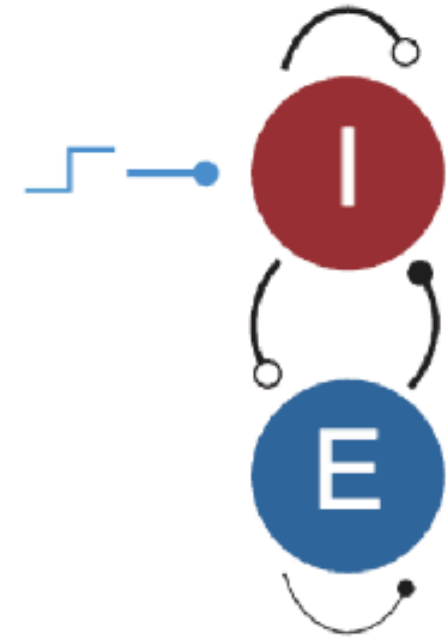
- * Full-rank response function
- Low-rank approximation
- Negative outlier
- Positive outlier
- Mean connectivity

[Shao et al 2024]



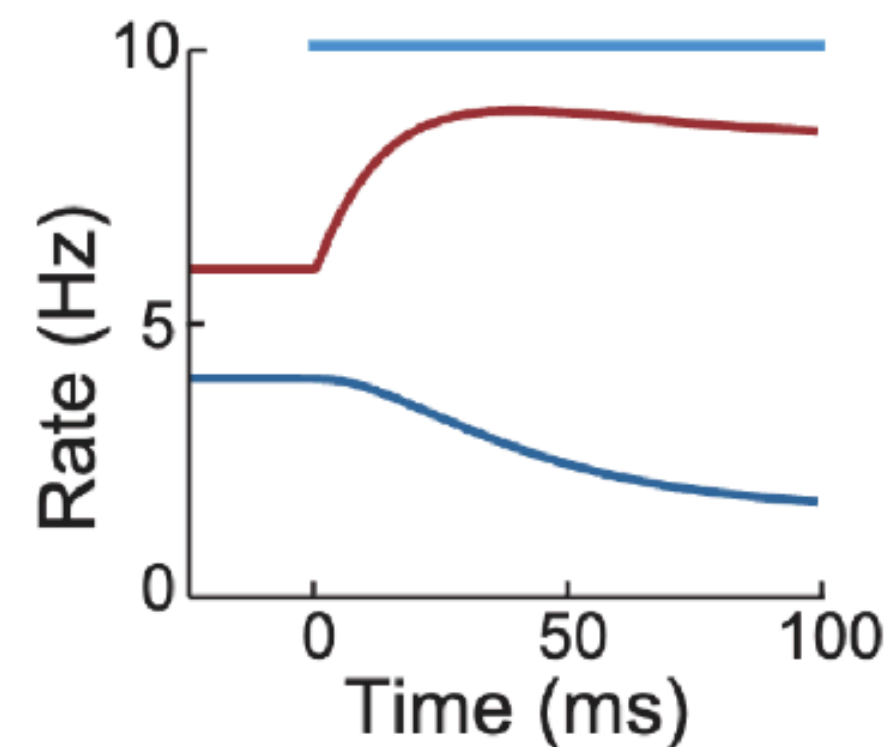
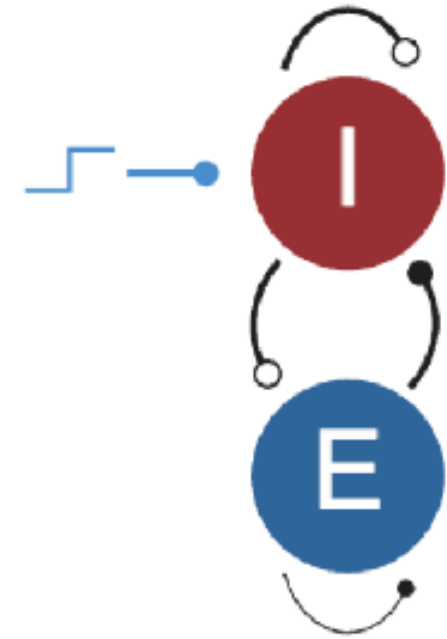
Strong deviations from predictions of mean connectivity

Paradoxical responses

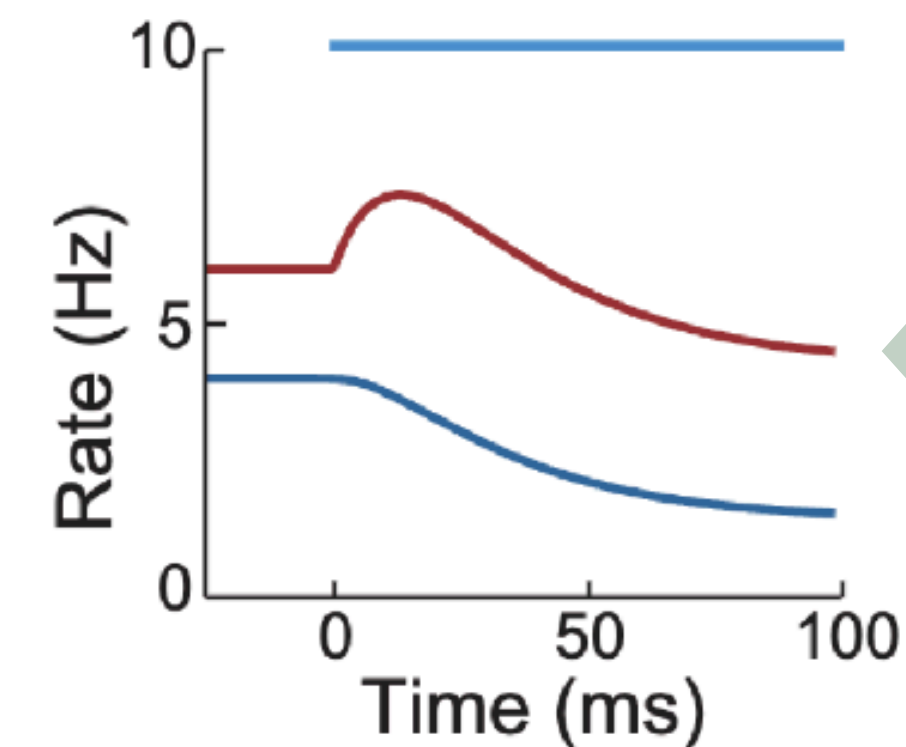
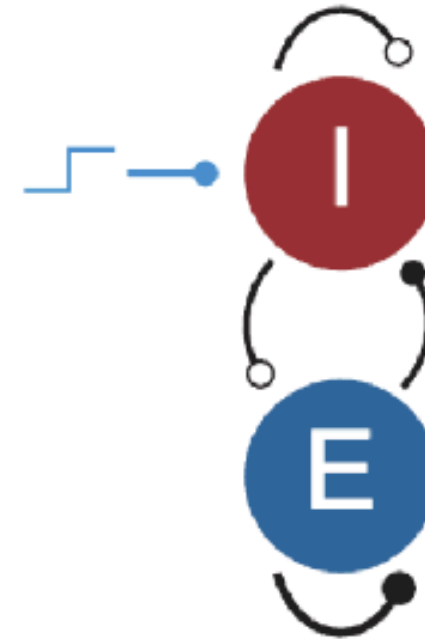


Paradoxical responses

Weak excitation



Strong excitation: Inhibition-stabilized networks



Paradoxical response

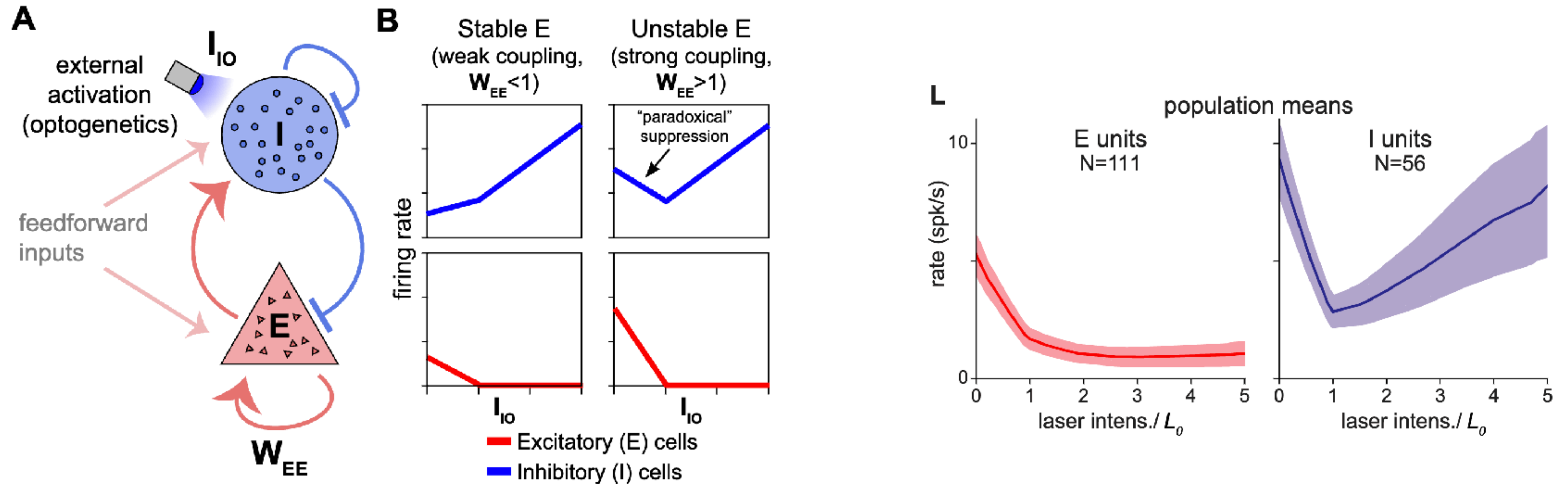
[Tsodyks et al 1997, Ozeki et 2009, Litwin-Kumar et al 2015, Miller & Palmigiano 2020]



Paradoxical responses as a signature of the operating regime in the cortex

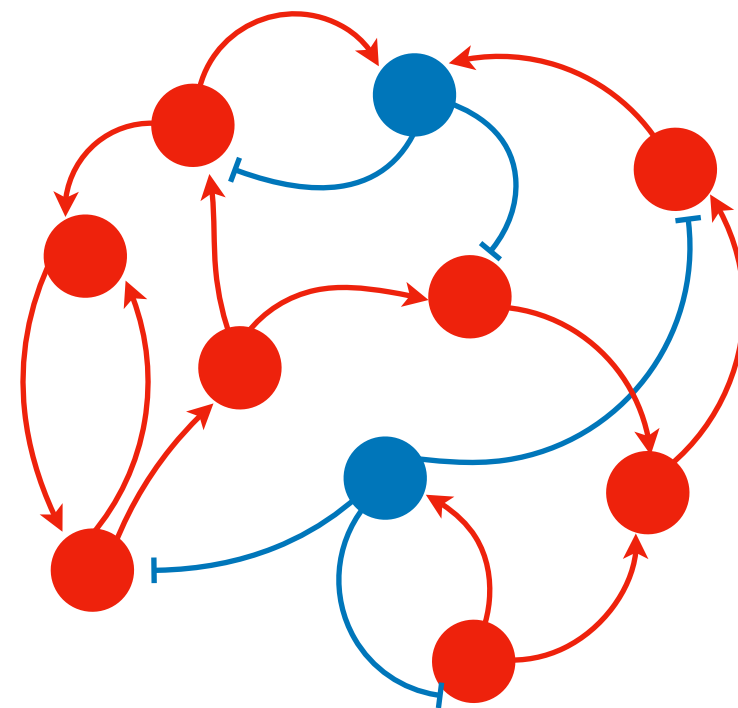
[Palmigiano et al 2020, Sanzeni et al 2020]

Paradoxical responses

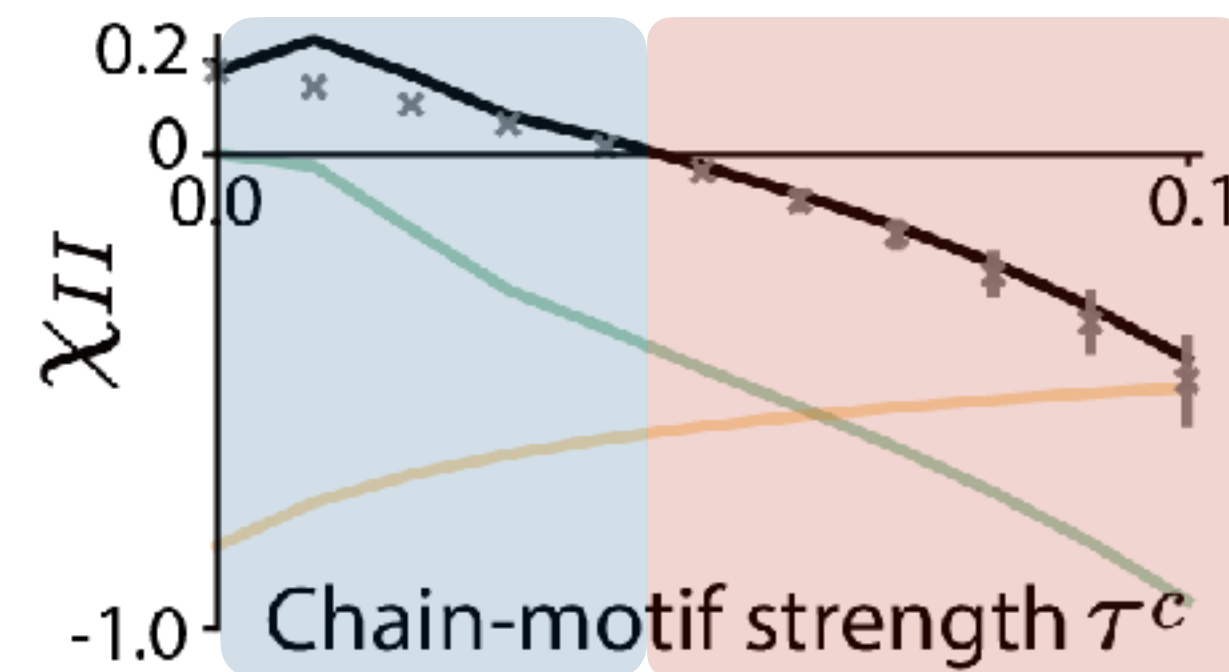


Paradoxical responses

Input to
Inhibitory cells



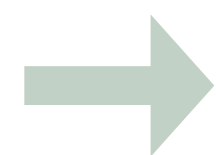
Average response, I cells



Non-paradoxical Paradoxical

- * Full-rank response function
- Low-rank approximation
- Negative outlier
- Positive outlier

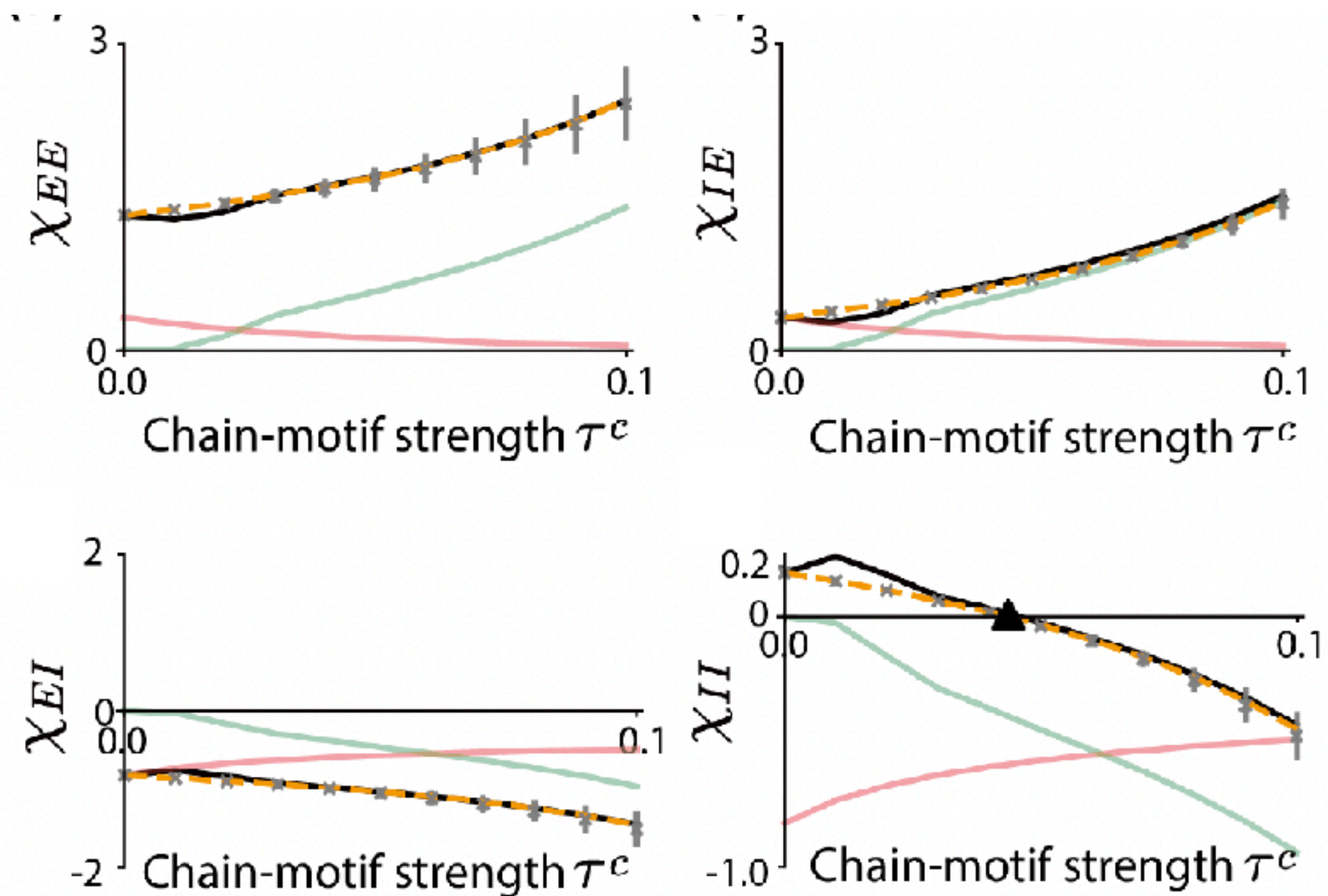
[Shao et al 2024]



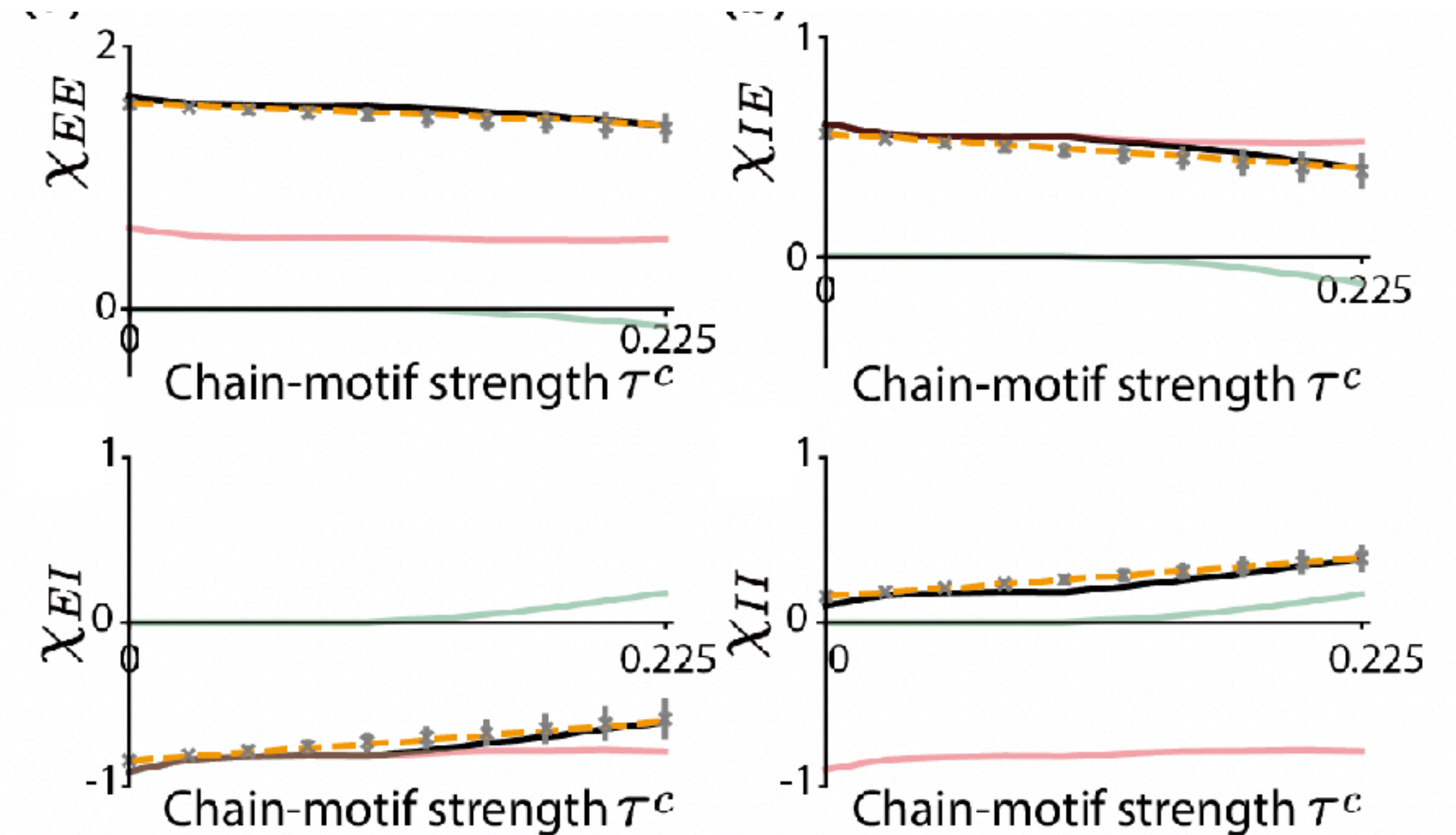
Strong deviations from predictions of classical circuit model

Paradoxical responses

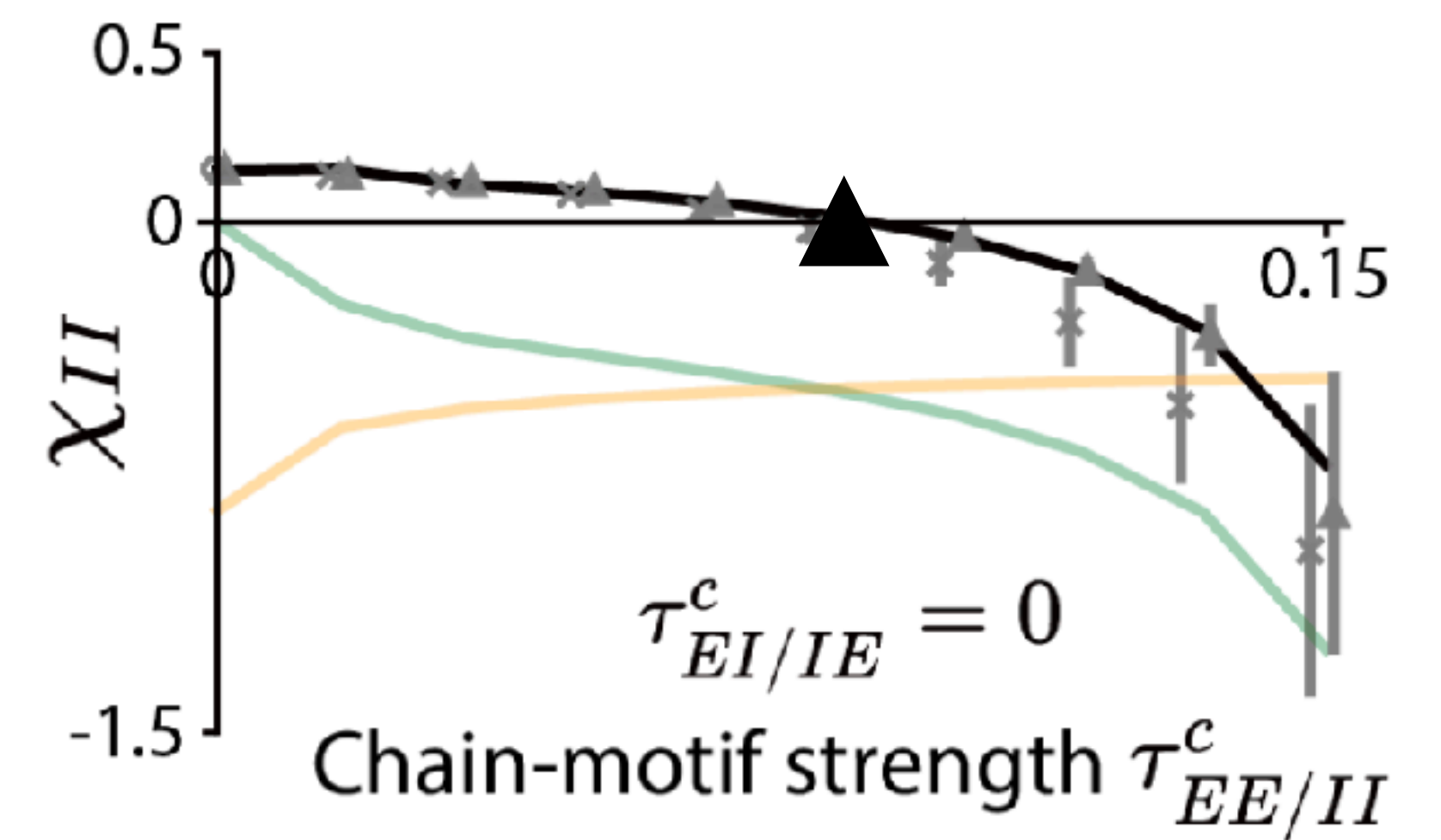
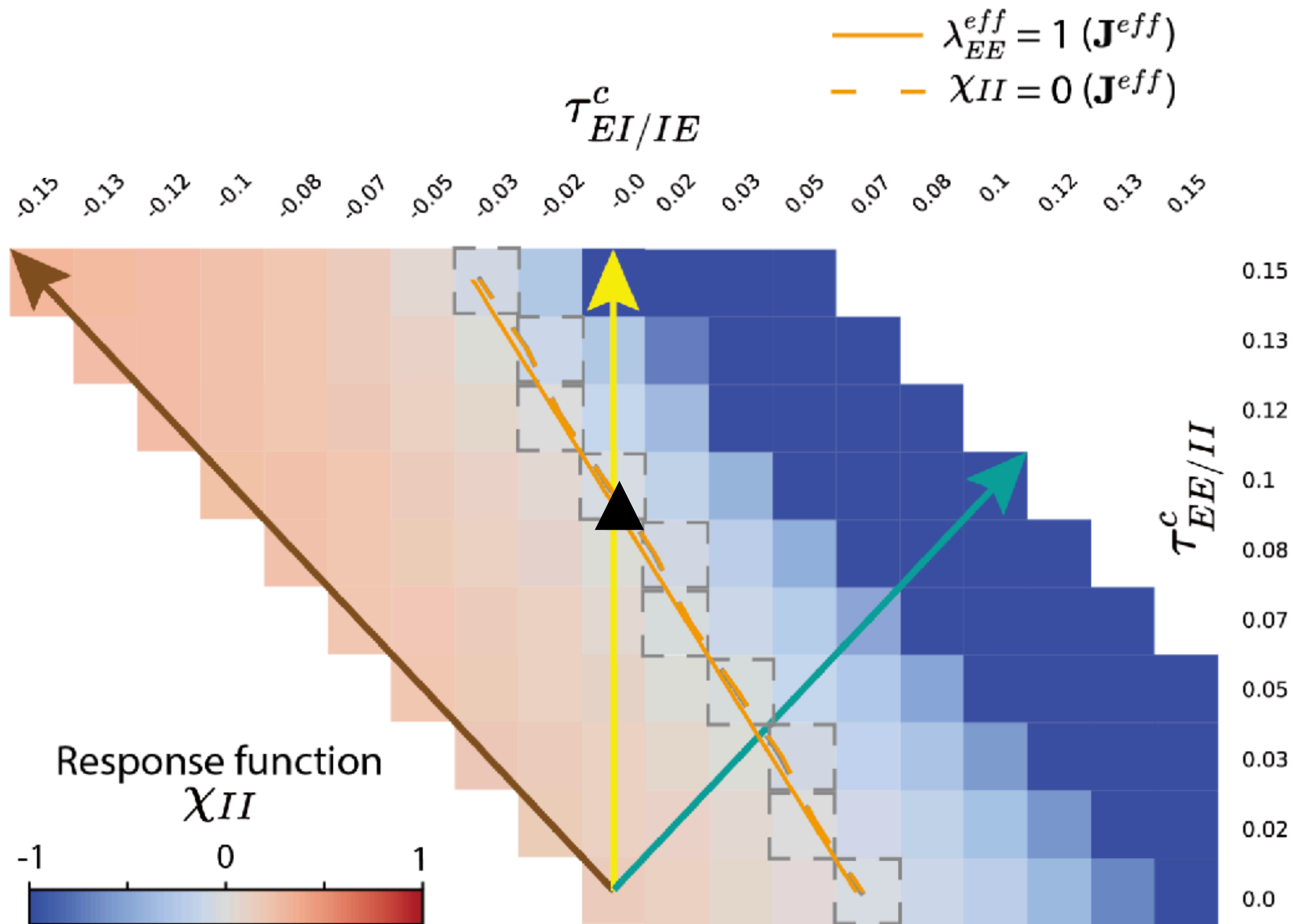
Homogeneous Gaussian



Heterogeneous Sparse



Paradoxical responses



Effective connectivity approximation

[K. D. Miller and A. Palmigiano, 2020;
Y. Hu et al., 2018]

Original Full-rank connectivity

$$J = J^0 + Z$$

$$[\chi] = \left[(1 - Z - \frac{1}{N} M_0 N_0^T)^{-1} \right]$$

Effective connectivity

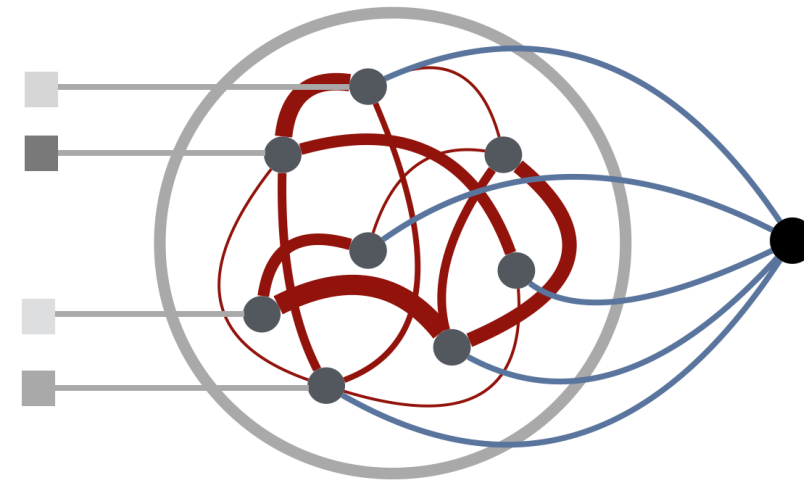
$$J^{eff} = J^0 + [Z^2]$$

$$\chi^{eff} = \left(1 - [Z^2] - \frac{1}{N} M_0 N_0^T \right)^{-1}$$

Woodbury matrix identity
(matrix inverse formula)

$$(1 - [Z^2])^{-1} + \sum_{k,l=0}^{\infty} [Z^2]^k \frac{M_0 N_0}{N} [Z^2]^l$$

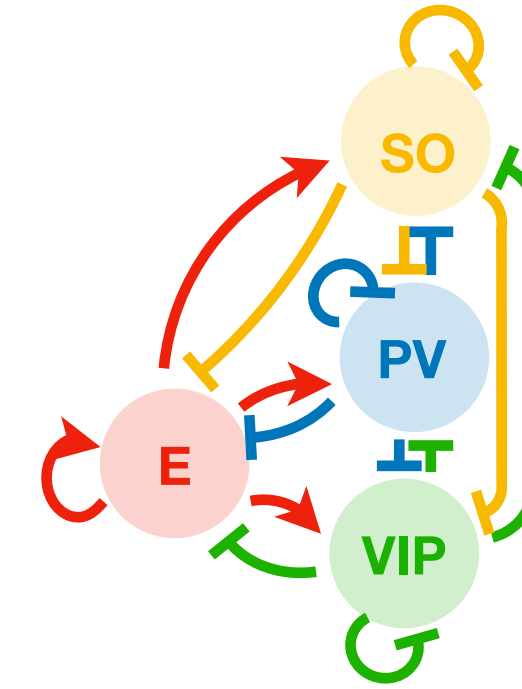
Networks



?



Circuits



1. When/how can we reduce a large network to a low dimensional models?
When low-rank structure is present in connectivity:
 - computational origin
 - **cell-type origin**
2. When do low-d units/factors correspond to average cell-type activity?
When they correspond to low-rank structure originating from cell-type organization
3. When does average cell-type connectivity correctly predict dynamics?
Strong deviations due to structure such as chain motifs

Acknowledgments

Ostojic's team



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Kai Chen (SJTU)

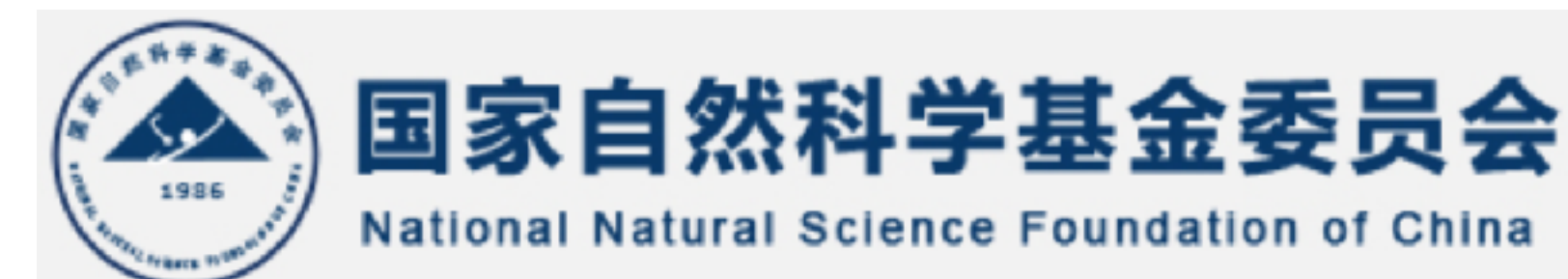
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Hang Zhang (PKU)

Bailu Si (BNU)

Hang Zhou (SIAT)



Heterogeneous Network Connectivity with Heterogeneous Motif Statistics

$$J_{ij} = J_{pq}^0 + \sigma_{pq} \left(\text{sgn}(\tau_{pq}^c) \sqrt{|\tau_{pq}^c|} \eta_i^{(q)} + \sum_{x=1}^P \sqrt{|\tau_{qx}^c|} \eta_j^{(x)} - \text{sgn}(\tau_{pq}^c) \sqrt{|\tau_{pq}^c|} \mu_{ij}^{(q)} + \sqrt{|\tau_{qp}^c|} \mu_{ji}^{(p)} \right. \\ \left. + \text{sgn}(\tau_{pq}^r) \sqrt{|\tau_{pq}^r/2|} v_{ij} + \sqrt{|\tau_{pq}^r/2|} v_{ji} + c_{rm,pq} y_{ij} \right),$$

$$c_{rm,pq} = \sqrt{1 - 2|\tau_{pq}^c| - |\tau_{qp}^c| - \sum_{x=1}^P |\tau_{qx}^c| - |\tau_{pq}^r|},$$

$$\lambda = \lambda_0 + \frac{\lambda_0 (N_E Z_E^2 + N_I Z_I^2)}{\lambda^2 - (N_E Z_E^2 + N_I Z_I^2)}.$$

$$[\mathbf{Z}^2]_{ij} = \alpha_E \sigma_E^2 \tau_{EE}^c + \alpha_I \sigma_I \sigma_E \tau_{IE}^c = Z_E^2, \quad j \in E, \quad \text{The nonzero solutions can be expressed as}$$

$$[\mathbf{Z}^2]_{ij} = \alpha_E \sigma_E \sigma_I \tau_{EI}^c + \alpha_I \sigma_I^2 \tau_{II}^c = Z_I^2, \quad j \in I,$$

$$\lambda = \frac{\lambda_0 \mp \sqrt{\lambda_0^2 + 4(N_E Z_E^2 + N_I Z_I^2)}}{2}$$